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ABSTRACT

The Forestry Commission of New South Wales has investigated the growth of plantation grown *Pinus radiata* D.Don since 1962. The development of stand growth / density relationships and the mathematical models formulated to simulate stand basal area growth since that time are detailed.

Four decades of experimental data have recently been analysed to formulate a stand basal area model that is based on a growth theory compatible with this species and that incorporates an innovative approach to :

- the determination of site potential index;
- accountability for previous thinning history in multiple thinning sequences;
- monitoring productivity and piece size for any thinning schedule.

The model predicts the stand basal area yield following any number of routine "thinnings-from-below" for a wide range of residual densities at any age within a forty year rotation. It also accommodates routine row removal at the time of the first thinning.

The method of model formulation and the statistical rigour of the analyses are documented. The model is tested for predictive basal area and basal area yield accuracy using data from measured experiments not included in the model formulation.

The complete data sets used for model formulation are available on application to the Forestry Commission of New South Wales.

INTRODUCTION

Pine plantation management requires accurate forecasts of future forest yield to be readily available, often decades or even a rotation, ahead of time. To this end, predictive modelling of the growth of *Pinus radiata* D. Don plantations in New South Wales has been an on-going process, gradually refined with time, as better analysis of more data has led to a greater understanding of growth processes. By 1985, a large body of data had accumulated from widespread and numerous experiments that had been established over four decades by Forestry Commission of New South Wales' researchers. At that time, very little of that data base had been analysed. To redress this shortcoming and to draw together the work done so far, this paper details and references the past work, reviews recent stand dynamics research findings and documents an innovative approach to the formulation of a *P. radiata* basal area growth model using a large proportion of the accumulated data.

Part I of this paper details the twenty three year history of *P. radiata* basal area growth investigation that began in 1962. Specifically :

(i) Development of stand growth / density relationships to determine optimal basal area thinning prescriptions;

(ii) Development of mathematical models which simulate stand basal area growth to forecast future yield accurately.

Part II reviews the development of the Response Increment method of analysis and the application of the results following the 1985 analysis of thinning trial data. The method showed that Möller's (1954) plateau growth theory is applicable to *P. radiata* and hence is a suitable basis for the 1988 basal area growth model. It defines and quantifies the stocking-densities of two critical stand-growth positions.

Part III analyses four decades of thinning trial data and describes the derivation, function, and statistical rigour of the 1988 *P. radiata* basal area model. It assesses site potential, establishes a growth theory basis to evaluate thinning prescriptions and tests the predictive accuracy of the model.

Appendix 1 presents a worked example showing how to apply the 1988 model, while Appendices 2 and 3 contain details of the data base analysed.

PART I. BASAL AREA GROWTH RELATIONSHIPS AND MODEL DEVELOPMENT 1962-1985

The thinning of pine plantations first began in New South Wales in the 1940's. To determine optimal thinning strategies, experiments were established in large plantation centres from 1951 onwards (Carter[1]). Generally, individual tree diameter, stand age and predominant height of the stand were recorded over a time period during which the stand stocking was progressively and variously reduced. Since then, as now, sawlogs were perceived to be the plantation end-product, experimental stands were thinned from below in an attempt to determine the minimum stand density that would accumulate maximum growth. This course of action followed a belief that the stand-growth to stand-density relationship for *P. radiata* plantations would be similar to the plateau pattern postulated for European conifer forests by Langsaeter (1941), and later expressed more fully by Möller (1954).

By 1962, Gentle, Henry and Shepherd (1962) had instituted a basal area system of stand regulation for New South Wales pine plantations. From an analysis of limited experimental data available at that time, they concluded that the optimal basal area for residual stands (i.e. the unique basal area which accrues maximum increment to the least number of trees) increased with increasing age. For an age range 12 to 14 years, they estimated that the optimal basal area range was 18 to 21 m^2 ha^{-1}, whereas for stand ages greater than 30 years, they estimated that the range had risen to 25 to 28 m^2 ha^{-1}. However they found little evidence of a "Möller plateau".

With the advent of computer technology in the 1970's, Cosco[2] analysed some of the continuing accumulation of thinning trial and continuous forest inventory plot data to formulate a basal area increment simulation model. By modelling the data according to the published growth equations of Buckman (1962), Clutter (1963), and Curtis (1967), he found that the following modified form of Curtis' equation in terms of age, site and relative density, provided the best data fit (Watt[3]) :

$$\log \mathrm{BAI} = -\,0.00950 - 0.00964\,\mathrm{A} + 0.05298\,\mathrm{SITE} - \frac{0.02575}{\mathrm{RD}}$$

[**BAI** = stand basal area increment (m^2 ha^{-1} yr^{-1}),

A = age in years,

SITE = stand predominant height (m) at age 20 years, and

RD = relative density

$= \dfrac{\text{existing basal area}}{\text{total possible basal area}}$]

All the independent variables of this relationship were reported to be "highly significant" and the model to have described 72% of the basal area increment data variation for 593 observations. This model was used to determine an optimal residual basal area schedule, consistent with a range of management objectives (Forrest[4]). Three examples are shown in Table 1 (a), (b) and (c). Although Forrest considered option (b) to be the most advantageous, a thinning prescription between (a) and (b) was adopted by the Forestry Commission at this time.

To examine plantation basal area growth theory further, Shepherd and Forrest (1973) analysed eleven years of growth data from one thinning trial, but also drew "on the experience and results gathered from other trials". They reaffirmed the earlier contention that the minimum stand basal area required to maintain maximum stand basal area increment increased with age and found that, irrespective of the thinning treatment imposed, basal area increment declined with age. Applying "third degree polynomial relationships" to the data they were unable to discern a "plateau-like" pattern, deciding rather that there was evidence of "a

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[1] Carter, P.R. (1981). Thirty years of thinning and spacing trials (1951-1981). For.Comm.N.S.W. Unpublished Report. 74 pp.

[2] Cosco, J.N. (1971). Growth and yield functions for *Pinus radiata* plantations in New South Wales. Symposium on forestry decision models. A.N.U. 13-14 July 1971. Unpublished 4 pp.

[3] Watt, A.J. (1970). Growth simulation of Pinus radiata plantations in New South Wales. Progress report 2. For.Comm.N.S.W. Unpublished Report. 4 pp.

[4] Forrest, W.G. (1971). A review of tree and stand growth in *Pinus radiata* plantations in New South Wales. For.Comm.N.S.W. Unpublished Report. 25 pp plus tables.

unique stand density .... where basal area is the greatest" (Figure 1).

Table 1. Optimal residual basal areas consistent with three Management Options (Forrest 1971).

(a) Maximum volume production on the minimum number of trees.

Stand Age (years)	15	20	26	33	40
Residual basal area ($m^2\ ha^{-1}$)	21	25	27	30	0

(b) A balance between total production size class distribution and discounted value returns.

Stand Age (years)	13	19	26	35
Residual basal area ($m^2\ ha^{-1}$)	11.5	16	23	0

(c) Maximum growth of final crop trees at maximum financial return.

Stand Age (years)	7	15	23	30
Residual basal area ($m^2\ ha^{-1}$)	7	11.5	14	0

In 1974, comparisons of actual and predicted basal area growth showed that the Cosco model was seriously underestimating basal area growth at plantation ages above 30 years. At that time, this was an important consideration since stands over 30 years old embodied a large component of future plantation yield, and it was expected that plantations would be managed over rotations longer than 40 years. Consequently, McMullan[1] chose an exponential model, which was a modified version of that devised by Sullivan and Clutter (1972). It simulated final basal area in terms of initial basal area, age and site without an increment term, and was fitted to a larger data set (2299 observations) as follows :

$$BA_2 = \exp\left[\frac{A_1}{A_2}\log BA_1 + c_1\left(1-\frac{A_1}{A_2}\right) + c_2\left(1-\frac{A_1}{A_2}\right)S\right]$$

[BA_1 = initial stand basal area ($m^2\ ha^{-1}$),
BA_2 = final stand basal area ($m^2\ ha^{-1}$),
A_1 = initial stand age (years),
A_2 = final stand age (years),
S = site index (i.e. dominant height (m) at age 20 years),
c_1 = 4.18908
c_2 = 0.0033759]

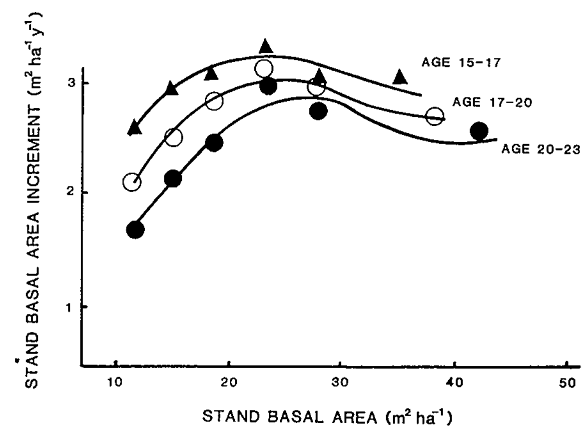

Figure 1. The relationship between stand basal area and stand basal area increment. The curves represent calculated third degree polynomials of best fit (Shepherd and Forrest 1973).

This model was reported to have described 77% of the data set variation. The coefficient of the first predictor variable (c_1) was reportedly "highly significant" but the coefficient of the site index variable (c_2) was "not significant at the 5% level". Later in 1974, an extra term was added to the exponent of the equation to account for the wide range of cumulative stand basal areas that

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[1] McMullan, M.J. (1974). Basal area growth function investigations. Unpublished notes. 6 pp.

normally result from variable thinning histories. This term was :

$$c_3 \left(1 - \frac{A_1}{A_2}\right) \left(\frac{P}{1+T^2}\right)$$

[P = proportion of basal area removed at thinning,

T = time elapsed since thinning, and

c_3 = 1.536]

Employing the Sullivan and Clutter model with the added thinning history term to analyse a data set of 1059 thinned stand observations accounted for 90% of the data variation. However, the coefficient (c_3) of the additional variable was "not significant at the 5% level". Even so, this basal area model including the thinning response variable, was ultimately chosen as the basal area simulation mechanism to be incorporated into a yield scheduling system (RADVAL) (McMullan 1979a). This system is part of a RADiata-Harvesting-OPtimisation computer package known as RADHOP, compiled at this time to optimise volume production with marketing constraints (McMullan 1979b). RADHOP was reported (Wilson 1979) to be formatted in similar fashion to Clutter's (1968) "MAX-MILLION" model and the "MASH" model of Gibson, Opie and Weir (1974).

Wilson & Watt (1976) used RADVAL to simulate a new set of optimal post-thinning basal area regimes for two options: (a) a market for pulpwood, and (b) no market for pulpwood, as shown in Table 2. In both cases the simulation advocated postponing the final felling for as long as possible, recommending that it take place no earlier than age 45 years. They agreed in principle with Forrest's earlier work that regimes involving few relatively intense thinnings are financially preferable to frequent and light thinnings.

Table 2. RADVAL simulated optimal post-thinning basal areas at given ages (Wilson and Watt 1976).

(a) Residual basal areas at given ages for a normal pulpwood market (Initial stocking 1483 trees ha^{-1}).

Stand Age (years)	13	22-25	30-32	37-40
Residual basal area (m^2 ha^{-1})	14.9	25.3	27.6	27.6

(b) Residual basal areas of given ages when there is no pulpwood market (Initial stocking 740 - 988 trees ha^{-1}).

Stand Age (years)	18-20	25-27	32-34	39-41
Residual basal area (m^2 ha^{-1})	22.9	25.3	27.6	27.6

However, as there was a need to establish wood-using industries, a shorter rotation of 35 years was estimated to be closer to optimum volume production and was adopted. Later, to improve the product mix and increase intermediate thinning yields, some subjective changes (McMullan[1]) were made in 1978 to Forrest's and Wilson's optimal thinning schedules. This "amalgam" of silvicultural compromises was accepted to be the preferred silvicultural thinning schedule and was quoted in New South Wales pine plantation management plans (Anon. 1981). Further minor subjective adjustments were made to these thinning schedules in 1983 (Anon. 1984, Anon. 1987), to increase early thinning yields (Table 3.)

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[1] McMullan, M.J. (1978). Aims and constraints to be considered in developing thinning regimes. In Proc. Pinus Plantation Management Conf., Tumut. 18-20 April, 1978. For.Comm.N.S.W. Unpublished.

Table 3. Preferred thinning schedules (a) Bathurst Management Plan (Anon. 1987) (b) Tumut Management Plan (Anon. 1984)

Stand Age (years)	Residual Basal Area ($m^2\ ha^{-1}$)			
	1st thinning		later thinnings	
	(a)	(b)	(a)	(b)
12		15		
13*	16	16		
14+	17	16		
15#	18	17		
16	19	17		
17	20	18	23	18
18	21	19	23	18
19*+	22	20	24	18
20	22	22	24	19
21	23	23	25	20
22	24	24	25	20
23	24	25	26	21
24#	25	25	26	21
25*+	25	26	28	22
26	26	26	28	22
27	26	27	29	22
28	27		29	22
29	27		29	22
30*+#	28		30	22
31			30	22
32			30	22
33			30	22
34			30	22
35^{cf}			30	22

*	indicates age of Tumut Management Plan preferred thinning.
+	indicates age of Bathurst Management Plan preferred thinning with good smallwood markets.
#	indicates age of Bathurst Management Plan preferred thinning with restricted smallwood markets.
cf	indicates preferred age of clear felling.

In 1980 Donovan[1] documented the RADHOP system, describing it as "a course of action which will meet wood supply commitments while optimising volume or value production". However, a compounding of errors in the basic prediction process, which became evident during a major yield scheduling task in 1983, led Bratby[2] to report instances of model volume over-estimation by up to 40% following logging operations in Albury Region. As a result, a review of the data sets and analysis methods used in formulating the RADVAL basal area model

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[1] Donovan, R.M. (1980). The RADHOP system. For.Comm.N.S.W. Unpublished Report. 28 pp plus appendices.

[2] Bratby, W. (1983). RADHOP exposed - bit by bit. For.Comm.N.S.W. Unpublished Report. 10 pp plus appendices.

was undertaken by Brack[1] in 1983. He reported that the model coefficients were calculated using data which had been subjectively selected from thinned and unthinned stands and that the model apparently had not been tested for bias or homogeneity of errors. Brack concluded that the form of the basal area model was not statistically sound and that predictions calculated from it were biased. This prompted a decision in 1984, to re-analyse a large portion of the ever-increasing *P. radiata* thinning trial data, to formulate a more accurate yield simulator. The work began in 1985 and is detailed in Parts II and III.

A brief summary of some management attitudes that were adopted following experimental data analysis between 1962 and 1984 is as follows :

(a) Rotation length :

30 years plus	Gentle *et al.* (1962)
30 to 40 years	Forrest (1971)[2]
45 years plus	Wilson & Watt (1976)
35 years	McMullan (1978)[3]

(b) Basal area simulator parameters. Optimal residual basal area was shown to be dependent on :

age	Gentle *et al.* (1962)
age, site index, relative density	Cosco (1971)[4]
age, site index, thinning history	McMullan (1974)[5]

c) Thinning :

The trend has been toward earlier, heavier and less frequent thinning.

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[1] Brack, C. (1983). RADHOP review - the growth simulations of RADVAL. For.Comm.N.S.W. Unpublished Report. 9 pp plus appendices.

[2] ibid.

[3] ibid.

[4] ibid.

[5] ibid.

PART II. DETERMINATION OF A STAND-GROWTH TO STAND-DENSITY RELATIONSHIP FOR *P. radiata* PLANTATIONS

Prior to the commencement of the 1985 analysis of growth data, an attempt was made to link the proposed growth model with an acceptable growth theory, something that forest modellers had been advocating for some time (Clutter 1963, Smith 1983). Also, as previous models had reported little detail of their adherence to linear regression assumptions, it was decided that the statistical rigour of the latest model would be more fully documented.

As to a model-growth theory link, Smith (1983) had suggested that, "For some species, the stand growth theory developed by Möller (1954), i.e. that the growth in cubic volume is little affected over a wide range of densities, seems appropriate, and its consequences in terms of basal area growth should be explored". This suggestion was adopted and subsequently a method of analysis which partitions stand growth (Horne, Robinson and Gwalter, 1986) was applied to the accumulated data to determine whether growth of an even aged forest species, such as *P. radiata* is compatible with the "Möller growth" theory. The analysis method, results and model implications are briefly discussed below.

THE METHOD OF PARTITIONING STAND GROWTH

The basal area increment of a residual stocking can be considered as a combination of two partial increments, i.e. time-dependent (or base) increment (I_B) which is defined as that accruing to any part or segment of the stand in the absence of any reduction in the total stand stocking, and competition-dependent (or response) increment (I_R) which is defined as the additional increment accruing to any part or segment of the stand as a response to a reduction in the total stocking. The total increment (I_T) can thus be represented as :

$$I_T = I_B + I_R$$

Horne and Robinson (1988) describe the method of partitioning data into stand-segments which enabled them to derive mathematical models for I_B and I_R as functions of residual stand stockings. These two models were then combined to form the increment (I_T) / density (N) curve.

RESULTS OF THE "RESPONSE INCREMENT" ANALYSIS

Horne and Robinson (1988), applied the Response Increment analysis method to nine replicated thinning experiments, with measurements spanning up to four decades and located in three major New South Wales plantation areas, to determine the increment relationship for *P. radiata.* They found that within the age range 10 to 35 years, the resultant pattern of basal area increment (I_T), calculated by adding partial increments I_B and I_R with respect to stocking density (N), consistently included a close approximation of a "Möller plateau" region, independent of location. In addition, Horne and Robinson postulated that two critical growth positions, derived from the response increment curve (I_R), could be estimated on the total increment curve (I_T). These two points were determined from the maximum response on the I_R curve (Point C) and the point of inflexion located to the right of the maximum response (Point P). The plateau edge "point P", is thus an objective estimate of the long-sought-after lowest stand stocking where the site maintains maximum stand increment. The maximum thinning response "point C" is a mathematically defined position that is characterised by a considerably lower stocking and consequently greater individual tree growth than that of point P, while retaining a high proportion of the point P maximum stand increment.

The determination of these growth positions, based on a growth theory which is demonstrably applicable to plantation *P. radiata* in New South Wales was accepted to provide a suitable theoretical basis on which to formulate a new growth model for this species (Figure 2).

IMPLICATIONS OF "RESPONSE INCREMENT" ANALYSIS FOR STAND BASAL AREA MODELLING

Three implications pertinent to basal area models can be drawn from this work.

First, critical point P basal area increment is an alternative or additional index of site potential.

Traditionally, the effect of site potential differences on stand growth has been incorporated into basal area growth models by a measure of stand height at a known age (Carron 1968, Watt[1]1970, Sullivan and Clutter 1972, Husch, Miller and Beers 1972, McMullan[2] 1974, Ferguson and Leech 1976, Bailey and Ware 1983). The reason for this is that tree height, unlike tree diameter, is independent of stand stocking over a wide range of values. Thus the site potential of stands with widely differing stockings can be estimated by a comparison of their respective stand heights at a given age. However, for basal area models, variation in the data due to site would be more directly accounted for if derived from stand basal area rather than indirectly derived from tree height. A direct link between site potential and basal area can be made for variably stocked stands if stand basal area increment, like height, can be shown to be independent of stocking for a wide range of values. Since this is the case, a simple site potential index can be obtained directly by modelling the stand basal area increment for all stockings within the "plateau" range, or more simply by modelling the increment of critical point P against stand age.

Second, the modelling of critical points P and C with age allows an evaluation of multiple thinning prescriptions in terms of the balance between individual tree growth (piece size) and maximum stand growth (productivity).

At densities above critical point P, individual piece size decreases without an increase in stand productivity. For densities falling below point P but above point C, piece size rapidly increases with only a relatively small corresponding decrease in stand productivity. For stands with densities below critical point C, piece sizes are larger still, but considerable stand productivity is forfeited. Thus critical point C has been considered an optimal growth position.

Third, estimation of residual basal area by accumulation of specific stand-segment basal area increments eliminates the problem of thinning history.

Initially similar, even-aged stands which undergo differing schedules of multiple thinnings and which ultimately have identical residual stockings at a mature age will have considerably different basal areas (Pienaar and Shiver 1986). As a consequence of this dependence on thinning history, models that predict basal area have to make provision for the number, age and intensity of past thinnings to account for the wide data variation encountered. Indeed the Sullivan and Clutter model (1974) used in the RADVAL simulator was altered to include a thinning history term. However, basal area derived by accumulating specific stand-segment basal area increment will progressively include the variations due to stand thinning history, without the requirement of a separate thinning history variable.

These concepts, in conjunction with a large proportion of the accumulated data were taken as the basis of the new model.

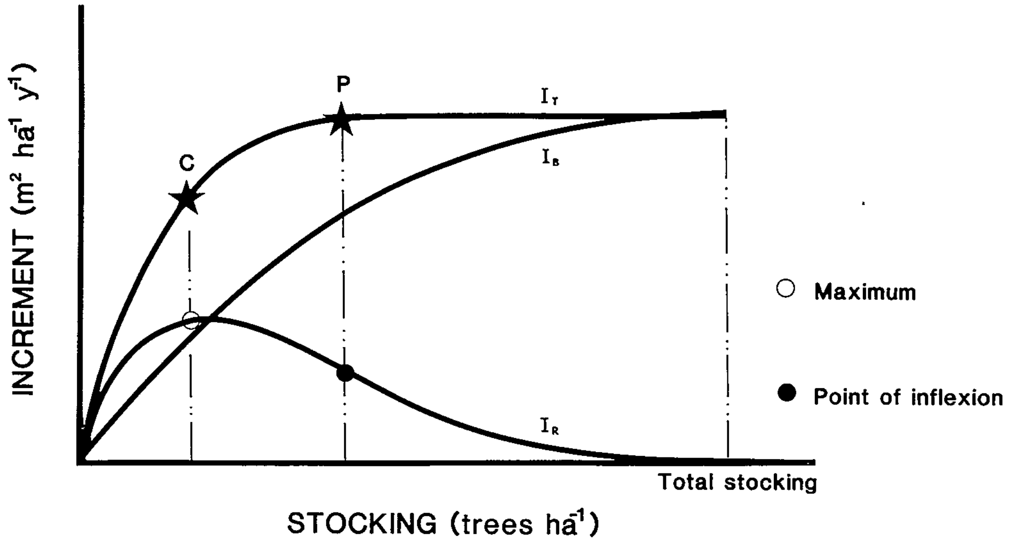

Figure 2. The relationship between stand basal area increment and stocking density (Horne and Robinson 1988).

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[1] ibid.

[2] ibid.

PART III. THE STAND-SEGMENT BASAL AREA MODEL

In 1986, a large proportion of the New South Wales accumulated experimental thinning trial data was analysed for the first time to formulate a growth model. It was intended that this model be based on more data than the previous models, be consistent with the growth theory determined for this species and include the various innovations that had resulted from the recent stand-increment to stand-density pattern studies. The model was required to predict the basal area of a stand following any number of thinnings, to any residual density undertaken (i) routinely from below, (ii) as an outrow thinning of given frequency or (iii) a combination of both, carried out at any age within a rotation of up to forty years. The resultant growth model is described below.

MODEL DESCRIPTION

The model is a stratified stand model in that it estimates the growth of any specified residual stocking or stand-segment within that residual stocking, at any time within the rotation. It estimates stand basal area increment based on stand stocking rather than stand basal area so as to incorporate the modelling advantages previously described, namely site potential determination and basal area accumulation that accounts for thinning history.

THE DATA BASE

Data is drawn from three New South Wales forestry regions - Albury, Bathurst and Eden. The model is formulated from measurements of nine major replicated thinning trials (shown in Table 4). These stands were planted over the period 1937 to 1960 and represent a range of residual stockings and basal areas resulting from a variable series of multiple thinnings, carried out "from below" at ages 3 to 28 years. The corresponding unthinned control treatments represent an initial planting density range of 1200 trees ha^{-1} to 1800 trees ha^{-1}. The experimental treatment plots were measured at intervals of one to five years with all trees being individually measured for diameter (dbhob).

The thinning method that generated the data is described as being generally "from below", with an additional removal of a given frequency of outrows at first thinning. Routine thinning "from below" usually includes removal of malformed, damaged and unthrifty trees of any size. Model application is thus limited to forest stands with past and proposed thinning regimes similar to that of the model data base.

METHOD OF APPLICATION

To apply the model to predict the growth of a given forest stand, the age and residual stocking of each proposed thinning must be chosen. These known stockings are designated as the residual stocking stand-segments (S). From the time of first thinning the model predicts the growth of these stand-segments together with that of the total stand stocking, for each period between thinnings, until the age is reached where following thinning, an individual stand-segment is scheduled to become the total stand stocking.

As an example of this procedure, consider how the model would deal with the following pre-planned thinning sequence :

Age (years)	Stocking (N) (trees ha^{-1})	
	Before thinning	After thinning
15	1000	300
20	300	200
25	200	100

Between the ages 15 and 20, the model conventionally predicts the basal area increment on the total stand (N = 300 trees ha^{-1}), but will also predict the increment on two specific stand-segments: (i) the future residual 200 stems within the current residual 300 total stem stand (S[1] = 200 trees ha^{-1}) and (ii) the future residual 100 stems within the current residual 300 total stem stand (S[2] = 100 trees ha^{-1}). From this increment information the model can estimate the basal area of the stand at age 20, both before thinning (N = 300 trees ha^{-1}) and after thinning (N = 200 trees ha^{-1}) and hence estimate the basal area yield of the thinning carried out at age 20 (BA(300) - BA(200)). This process is then repeated between the ages 20 and 25 years, incrementing the new total stand (N = 200 trees ha^{-1}) and the stand-segment next scheduled to be a residual stocking (S[2] = 100 trees ha^{-1}) to determine the thinning yield at age 25 (BA(200) - BA(100)).

Table 4. Details of the nine major thinning trials forming the basis of the 1988 stand-segment basal area model.

Year of planting	Forestry Region	Location (Code on Figures)	Description at trial planting	Treatments (T) Replicates (R)	Range of Measurement Ages
1937	Eden	Bondi (E)	Production thinning	4T,.4R	15 - 28
1941	Bathurst	Gurnang (F)	Production thinning	3T, 4R	12 - 35
1945	Albury	Red Hill (A)	Pre-commercial thinning	5T, 4R	11 - 22
1949	Bathurst	Sunny Corner (G)	Production thinning	7T, 3R	14 - 28
1954	Albury	Buccleuch II (B)	Pre-commercial thinning	4T, 5R	9 - 12
1957	Albury	Buccleuch III (I)	Variable spacing	7T, 3R	10 - 18
1959	Bathurst	Vulcan (J)	Variable spacing	7T, 3R	10 - 15
1960	Albury	Buccleuch I (C)	Correlated curve trend	8T, 3R	10 - 23
1960	Albury	Green Hills (D)	Correlated curve trend	8T, 7R	12 - 21

MODEL FORMULATION - METHOD OF ANALYSIS

The model equations have been formulated by the method of least squares regression (Draper & Smith 1981), with backward elimination of non-significant predictor variables (5% significance level) and with consideration given to the contribution of each predictor variable to the coefficient of determination (R^2). For each model equation determined in this way, the number of data points (n), the calculated coefficient of determination (R^2) and the statistical probability (p)[1] are given in Table 5. The calculated probability has been given in preference to the more conventional levels of significance for the reasons set out by Warren (1986).

For these models to be statistically credible, the degree to which the linear regression assumptions have been met should be evaluated and the implications discussed. For the linear regression model

$$Y_i = a + \Sigma b_j X_{ij} + \varepsilon_i$$

the assumptions are that the residuals (ε) should be independent, homoskedastic and normally distributed with mean zero (Draper & Smith

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[1] p values given are those resulting from testing the null hypothesis that a predictor variable's coefficient = 0.

Table 5. Coefficients of model equations 1(a), 1(b), 2(a), 2(b), 3 and 4.
Table shows : equation coefficients (a, b, c, d, e); number of data points (n); coefficient of determination (R^2); probability values resulting from testing H_o:a=b=c(=d=e)=0 shown as (p>F); probabilities from testing H_o:residuals are normally distributed, shown as p<W (Shapiro-Wilk test) or p>D (Kolmogorov test).

Equation		Coefficients									
		a	b	c	d	e	n	R^2	p>F	p<W	p>D
I(a)	Increment of critical point "P"	+3.2260	-0.7851	-0.1176	.	.	20	0.858	0.0001	0.071	.
I(b)	Increment of critical point "C"	+3.0235	-0.7918	-0.1030	.	.	20	0.713	0.0001	0.704	.
II(a)	Stocking of critical point "P"	+7.9735	-0.5571	+0.2637	.	.	31	0.700	0.0001	0.577	.
II(b)	Stocking of critical point "C"	+7.4199	-0.6277	+0.3081	.	.	31	0.638	0.0001	0.964	.
III	Basal area increment of stocking-segment (full data set)	-0.2745	-0.4145	+0.7509	-0.3162	-0.0459	3464	0.906	<0.0001	.	0.022
	(independent data)	-0.2879	-0.4159	+0.7573	-0.3190	-0.0436	586	0.901	0.0001	0.447	.
IV	Basal area of stocking-segment at first thinning	-2.4643	+0.8936	-0.00025	-0.5312	+1.0623	3632	0.995	<0.0001	.	<0.01

1981). To test for homoskedasticity, the residuals have been plotted against the predicted values and the distribution visually assessed for randomness. To test for normality, the residuals have been assessed by the Shapiro-Wilk test (W) for a sample size less than 2000 observations and the Kolmogorov test (D) for a sample size over 2000 observations, as calculated by SAS (UNIVARIATE procedure, Anon. 1985).

SITE POTENTIAL CATEGORY

Important model variables such as, pre-thinning basal area, post thinning basal area and annual basal area increment, are all dependent on the site potential of the stand. Thus it is necessary to model the range of site potential categories inherent in the data.

Site index categories are commonly determined by fitting polymorphic or anamorphic curves and subjectively dividing the spread of data into site classes based on arbitrary criteria, e.g. 10 m height classes at age 20 (Carron 1968). Figure 3 shows that the stand plateau increment to age data fall into two distinct groups. Thus the spread of the data was used as the primary criterion for site category definition, inferring two different site categories. Inspection of Figure 3 suggests that an additional site potential category can be positioned between the two previously defined categories. Two further site categories can be recognised above and below the range of observed experimental data (Figure 4).

In this instance, the width of the nominal 95% confidence interval around the upper mean regression line at age 12 was calculated and chosen as the width of site potential classes.

The advantage of this procedure is that it allows the degree of variation shown by the critical point P data to determine the number of site categories inherent in that data. In this case the data variation delineated five increment ranges (two observed, one interpolated and two others: site categories Q1 to Q5 in Figure 4), such that for model equations I, III and IV :

K	=	-0.5	for site category Q1
		0.0	for site category Q2
		+0.5	for site category Q3
		+1.0	for site category Q4
		+1.5	for site category Q5

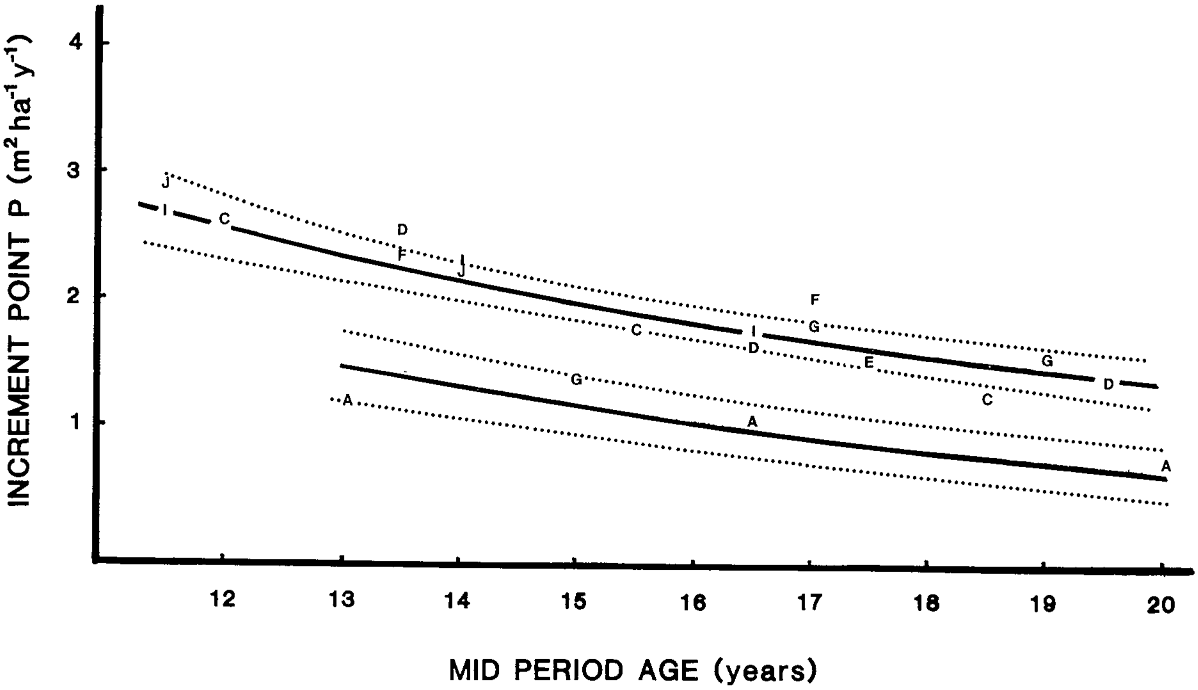

Figure 3. The relationship between point P (taken to represent stand plateau basal area increment) and mid-period age for experiment locations A-J (Table 4), showing nominal 95% confidence intervals for the mean regression (Model Ia). The lower line represents data from Red Hill locality (symbol A).

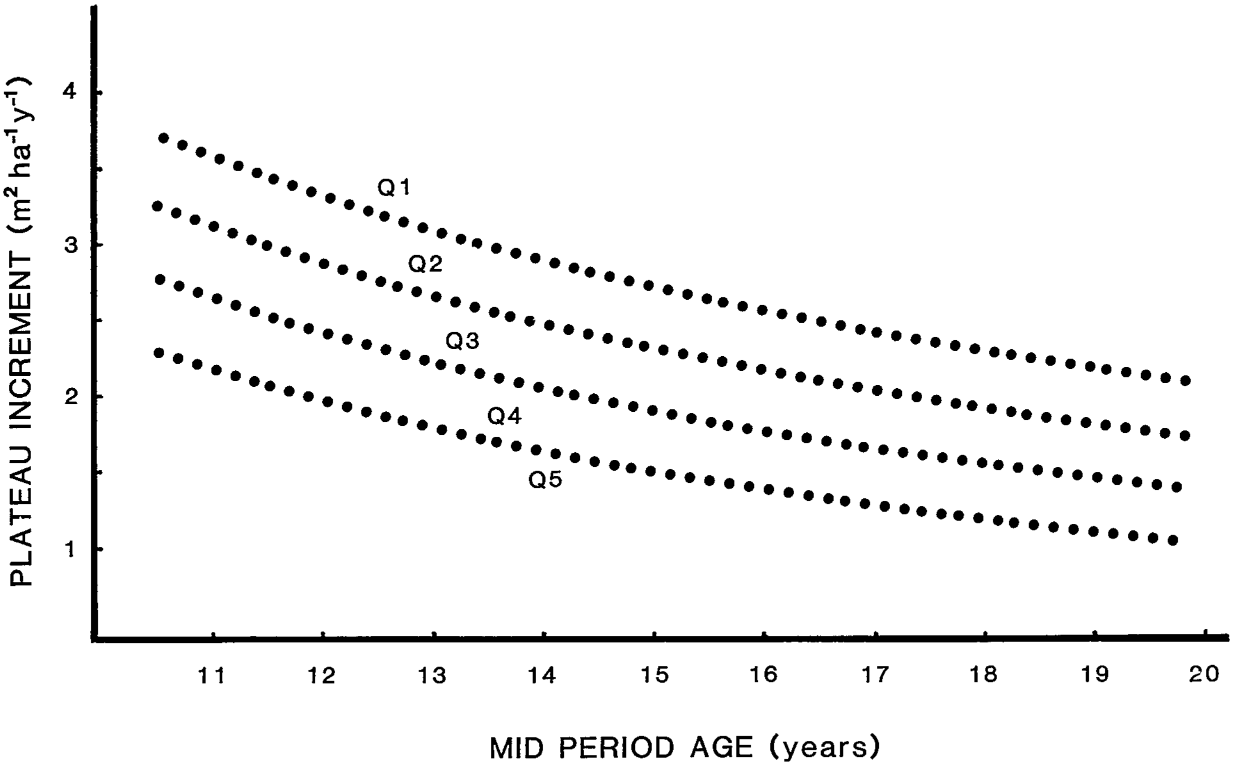

Figure 4. Site potential categories based on plateau increment and stand age.

Thus the site potential category of any stand with a stocking equal to or greater than N(P), where stand increment is at plateau level, may be determined from Figure 4 given a representative estimate of stand basal area increment and the mid-period age to which it refers. If the stand stocking is less than N(P), i.e. the stand increment is below plateau level, the site potential category can be determined from a comparison of the measured stand increment with a table of calculated increment of critical points P and C for site categories Q1 to Q5. The method of calculating a table of increments is illustrated uner Step 1 of Appendix 1.

MODEL EQUATIONS

1. *Critical Points P and C*

As Figure 3 shows, a linear regression of the critical point P increment on age data (taken from Horne and Robinson 1988 - see Appendix 2) for which the age of the first measurement of the period was at least 10 years and the mid-period age was not greater than 20, resulted in two statistically separate increment strata. Since it is intended to index site potential to the stand plateau basal area increment I(P) from measurements taken between the ages of 10 and 20 years for reasons previously explained, these strata indicate that the data represent two separate site potential categories. The form of the regression shown in Figure 3 for point P is :

$$\log I(P) = a + b \log A + c K \log A \qquad ...... I(a)$$

and similarly for point C :

$$\log I(C) = a_1 + b_1 \log A + c_1 K \log A \qquad ...... I(b)$$

[A = mean increment period age between 10 and 20 years,

K = site potential index such that K = 0 for the upper stratum and K = 1 for the lower stratum shown in Figure 3, and

a, b and c and a_1, b_1 and c_1 are constants.]

Table 5 shows that models I(a) and I(b) describe 86% and 71% of the critical point increment data variation respectively. The distribution of the residuals for each model (shown for equation I(a) in Figure 5) are considered to be suitably random and the distribution of the residuals is not shown to be other than normal (Table 5). As the data for both models were collected from only 9 locations, the data set includes some time-sequence points which will have non-independent residuals. However the very low p values ($p \leq 0.0001$ for H_o:a=b=c=0) suggest that the relatively small degree of non-independence is of little consequence.

From Table 5 it can be seen that models II(a) and II(b) describe 70% and 64% of the critical point stocking data variation respectively. The distribution of the residuals (shown for equation IIa in Figure 6) is suitably random and is not shown to be other than normal. Again, the very low p values ($p \leq 0.0001$ for H_o:a=b=c=0) suggest that the relatively small degree of non-independence is of little consequence.

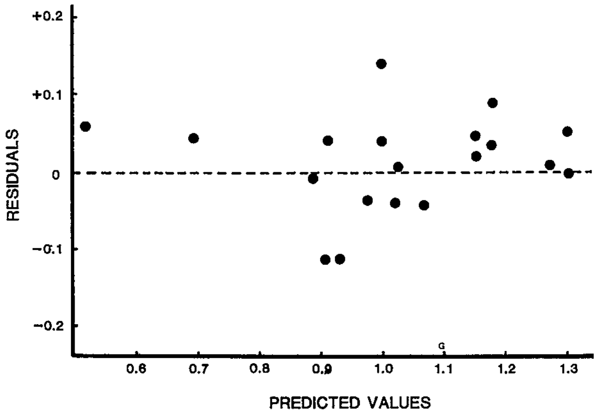

Figure 5. Plot of critical point P increment (Model IIa) residuals against the logarithm of predicted point P basal area increment. The pattern of the residuals does not indicate model bias.

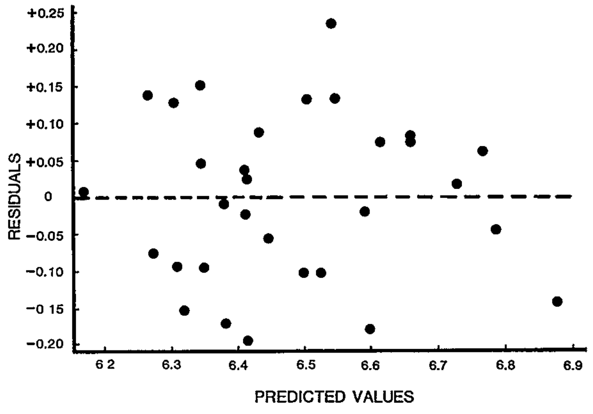

Figure 6. Plot of critical point P stocking (Model IIb) residuals against the logarithm of predicted point P stocking. The pattern of the residuals does not indicate model bias.

In the same way, fitting a linear model to the critical point stocking data (N(P) and N(C)) shown in Appendix 2, for which the age of the first measurement of the period was at least 10 years, with respect to age also resulted in two significantly different strata (using a 5% significance level). However, in this case the upper stratum corresponded to one particular locality, Bathurst Region, indicating that Bathurst sites reach maximum increment at higher minimum stockings than the Tumut and Eden localities. The form of the regression for critical point stocking is :

$$\log N(P) = a + b\,(\log A) + c\,L \qquad \text{......II(a)}$$

$$\log N(C) = a_1 + b_1\,(\log A) + c_1\,L \qquad \text{......II(b)}$$

[L = 0 for locations other than Bathurst region, and

L = 1 for Bathurst region.]

By plotting the increment and stocking for critical points P and C as predicted from models I and II, it is possible to approximate the complex Weibull increment / density relationship (i.e. the sum of the I_B and I_R Weibull models) shown earlier. Hence the stand increment can be simply approximated for any stand stocking, at any age, site potential and location.

2. *The Basal Area Increment of Stand-segments*

The principal and key component of the model is the estimation of future increment for specific stand-segments. The size of a stand-segment for analysis can be any number of trees per hectare in theory, however the lower stocking limit is set in practice by the size of the sample plot. In this analysis, the segment stockings have usually been set as multiples of the 100 largest dbhob trees per hectare for each treatment plot, and are re-ranked at each age (i.e. the largest 100 trees ha^{-1}, the largest 200 trees ha^{-1} and so on).

The experimental data base (details in Appendix 3) used to model increment (I(S)) has been sub-divided into age (A), thinning treatment residual stocking (N), and stand-segment (S) for each location and replication. Using these parameters, a linear regression model of the following form was developed from these data :

$$\log I(S) = a + b \log N + c \log S + d \log A + e K \log N \quad \text{...... III}$$

[a, b, c, d and e are constants.]

For a large data sample (3464 observations), model equation III described 91% of the data variation (Table 5). The excellent distribution of residuals with respect to predicted values (Figure 7) attests to the overall good fit of the model. Although there is some non-independence of residuals and their distribution is shown to be non-normal by a Kolmogorov test ($p = 0.022$), it is considered that the excellent residual pattern and the low probability figure ($p < 0.0001$ for H_o:a=b=c=d=e=0) diminishes the effect of these factors. However, as an added precaution to test this assumption, a re-analysis was carried out using a reduced but independent data set (586 observations) that were selected randomly from the larger set (West, Ratkowsky and Davis, 1984). The re-fitted model coefficients and R^2 value were found to be almost identical (Table 5) with those of the total data set. Moreover, the residuals were now demonstrably independent and normally distributed.

3. Initial Basal Area at the time of First Thinning

As a basis to predict future basal area following a sequence of thinnings, the initial basal area, ($BA_o(S)$) of any stand-segment (S) at the time of first thinning, must be determined. This can be done by simply measuring the basal area of the specific stand-segments at the time of first thinning or by predicting them from a linear regression model of the following form :

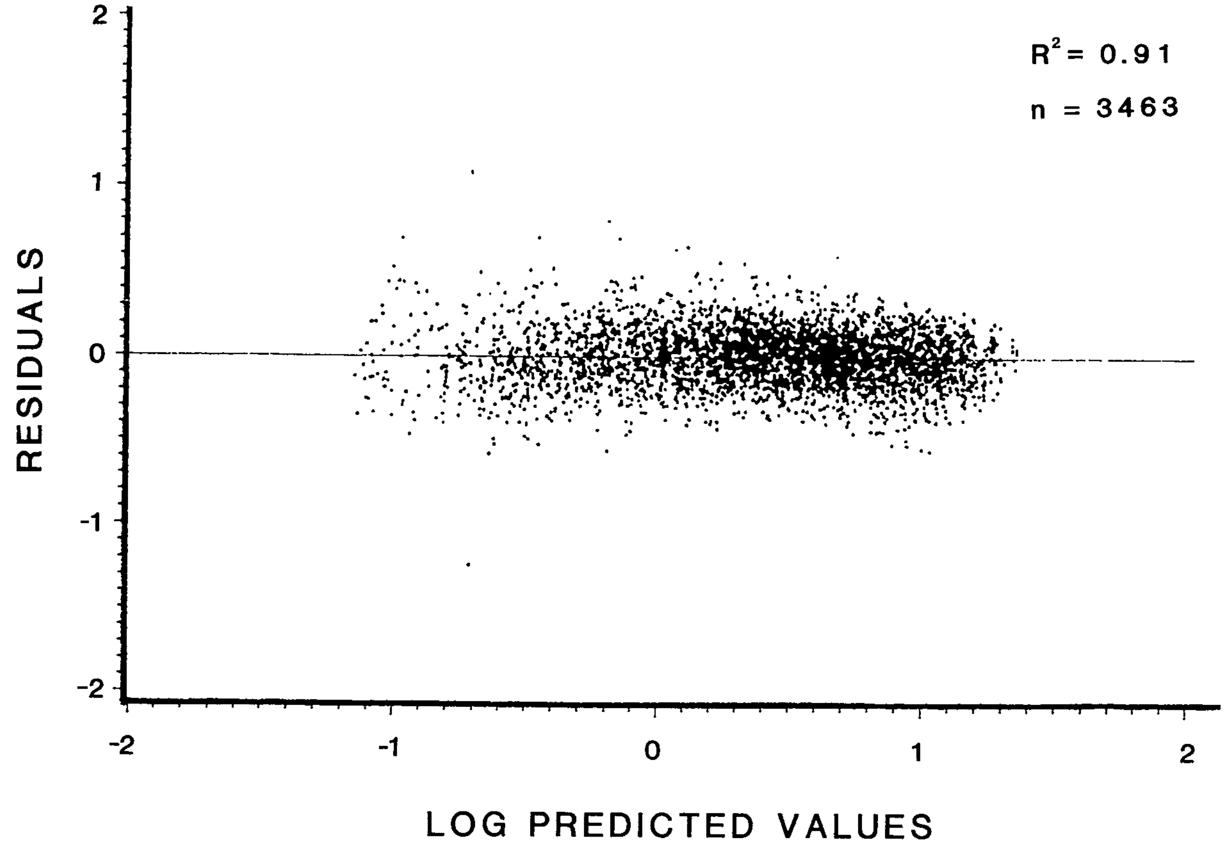

Figure 7. Plot of the stand-segment model III residuals against the logarithm of predicted stand-segment basal area increment. The pattern of the residuals does not indicate model bias.

$$\log BA_o(S) = a + b \log S + c\,S + d \log N + e \log BA_o(N) \quad \ldots\ldots \text{IVa}$$

[$BA_o(N)$ = total basal area of the stand prior to first thinning.]

For a large unthinned data set of 3632 observations (details in Appendix 3), model equation IVa describes 99.5% of the data variation (Table 5). Although the distribution of residuals appears suitably random, the data are not independent with respect to S, and the distribution of the residuals is not shown to be normal. Again, it is considered that the random distribution of the residuals and the low probability figure ($p<0.0001$ for H_o:$a=b=c=d=e=0$) diminishes any effect of the non-normality of residuals.

In routine practice, New South Wales first thinnings include a systematic row removal ("outrows") for machine access in addition to a silvicultural thinning from below. To account for this, model equation IVa can be re-expressed :

$$BA_o(S) = \frac{n-1}{n}\left[\exp\left(a + b \log \frac{n}{n-1} S + c \frac{n}{n-1} S + d \log N + e \log BA_o(N)\right)\right] \quad \ldots\ldots \text{IVb}$$

[n = frequency of outrows at the first thinning.]

STAND BASAL AREA PREDICTION

As the increments (I(S)) accruing between thinnings to specific stand-segments (S) are predicted from model III, then the basal areas of the future residual stockings (chosen to be equivalent to the specific stand-segments), can also be predicted. This is done by summing the basal area at first thinning with the basal area increments of each specific stand-segment accrued thereafter, viz -

$$BA_n(S) = BA_o(S) + \sum_{i=1}^{n} I_i(S)\ T_i$$

[$BA_n(S)$ = basal area of a given stand-segment of stocking S following n thinnings of the stand,

$BA_o(S)$ = basal area of the same stand-segment (stocking S) at the time of first thinning of the stand,

i = thinning sequence number (i.e. 1st thinning, 2nd thinning, etc.),

$I_i(S)$ = annual basal area increment of the same segment (stocking S) between the i^{th} and the $(i+1)^{th}$ thinning,

T_i = time in years between the i^{th} and the $(i+1)^{th}$ thinning.]

AN OPTIMAL GROWTH THINNING SCHEDULE

Maintaining the stand at the stocking or basal area which accrues optimal increment (i.e. Point C) has been shown to have the following advantages, viz. fast individual tree growth because of relatively low stand stocking and retention of a high proportion of maximum stand growth.

The model calculates the following "point C" residual basal areas and stockings for a Q2 site potential stand, if thinned at the preferred ages shown in Table 3 for (a) Bathurst Management Area and (b) Tumut Management Area :

(a) Age (years)	14	19	25	30
Basal area ($m^2\ ha^{-1}$)	18	25	33	39
Stocking (trees ha^{-1})	435	358	301	268

(b) Age (years)	13	19	25	30
Basal area ($m^2\ ha^{-1}$)	13	22	29	34
Stocking (trees ha^{-1})	334	263	221	198

It is clear that the third and fourth thinning residual basal areas shown above are considerably higher than those recommended in the preferred schedules of the Management Plans (Table 3) and so the optimal point C recommended residual basal area would yield lower productivity and smaller logs at these later thinning ages. However, from a growth perspective (as distinct from marketing or economic considerations), application of the preferred management plan prescriptions which are much lower than critical point C will mean considerable stand growth loss at later ages.

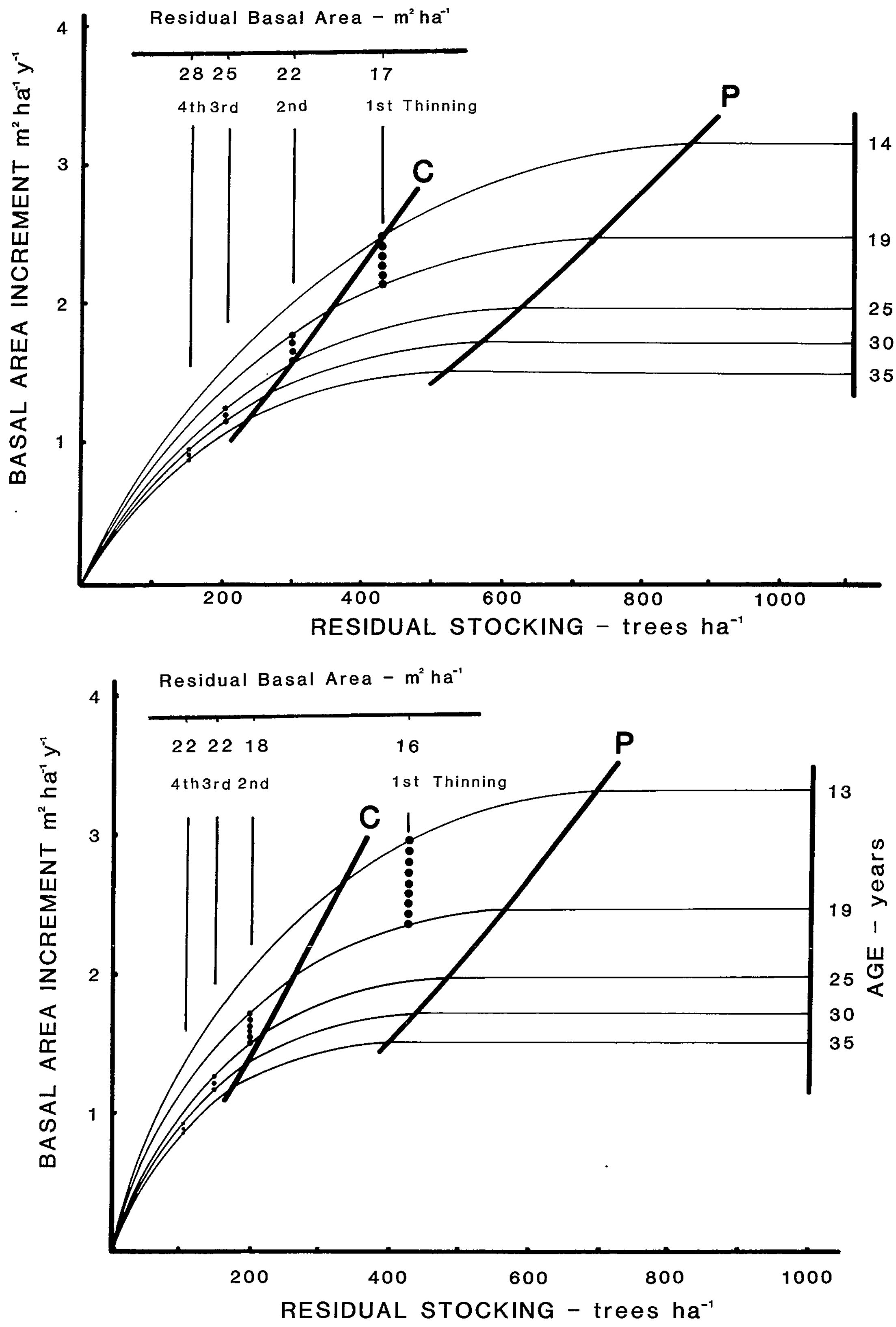

Figure 8. Stand increments of critical points P and C, predicted by the model for site potential Q2, are shown as heavy lines for a range of stand ages and stockings. To evaluate increment losses following implementation of prescribed thinning schedules in Bathurst Management Plan (top) and Tumut Management Plan (bottom), the predicted between-thinning range of stand growth is shown as a row of dots. The result is that in relation to optimal point C, excessive stand growth is shown to be forfeited from the time of second thinning using these management plan schedules.

Figure 8 shows the critical stand increments and residual basal areas as predicted by the model for both points P and C in relation to stand age and stocking. The relative positions of the New South Wales preferred thinning schedules for Bathurst (top) and Tumut (bottom) have been plotted in and show that a degree of stand growth has been forfeited from the time of second thinning. In this way, the model can be used to inform the forest manager about the stand growth efficiency resulting from changes contemplated for any proposed logging sequence.

TESTING THE MODEL

The model is intended to predict the mean basal area growth of relatively large areas of even aged *P. radiata* plantations in response to routine thinning from below given an unbiased and representative number of sample measurements from the area to be thinned. As such, it does not lend itself to accurate prediction of the growth of a few small localised experimental treatment plots from within a plantation area. Notwithstanding, the model has been tested (in the same manner shown in the example of model application given in Appendix 1), by comparing the model predicted growth to the measured growth for all the thinning sequence data from two measured thinning experiments that were not included in the model formulation (Table 6).

The measured experimental thinning sequences selected to test the model predictions are taken from experiments BT11471, Sunny Corner State Forest in Bathurst Region and AT11170, Green Hills State Forest in Albury Region. These experiments were chosen because the data, (i) were not included in the growth model formulation, (ii) include sequential thinning carried out to a range of intensities, over a long period of time with a mixture of "thinning-from-below" and "outrow" treatments, and (iii) are from experiments located in the two major New South Wales *P. radiata* plantation areas which are some 400 km distant from each other.

The Sunny Corner experiment was measured from age 14 to 34 years. It compares the stand growth of two thinning intensity regimes, spread over two separate thinnings and includes examples of routine thinning-from-below, routine thinning-from-below plus third row thinning and third row thinning. The Green Hills experiment was measured from age 15 to 32 years. It compares the stand growth of five different thinning intensity regimes, spread over five sequential thinnings which were undertaken routinely from below.

The measured growth of experimental plots from Sunny Corner and Green Hills State Forests has been compared with the model prediction for a total of ten individual thinning sequences. Table 6 summarises the comparison of measured and predicted basal area and yield as a percentage of predicted value to measured value for the ten sequences. Generally there is close agreement between the predicted and measured basal areas.

1. Prediction of individual stand basal areas before and after thinning.

For individual stand basal area predictions at given ages, Table 6 shows that for the Sunny Corner sequences, from 25 predictions 4 were within 2% of the measured values, 16 within 5% and all were within 10%. For Green Hills, from 48 basal area predictions 19 were within 2% of the measured values, 34 were within 5% and 47 were within 10%.

In the case of Sunny Corner individual basal area predictions, the model tends to overestimate (heavy type) throughout by about 4% (Table 6). Although the average individual basal area predictive ability of the model is excellent (99-100%) in the case of Green Hills, the model has a tendency to overestimate (heavy type) stand basal area early in the rotation and underestimate (lighter type) basal area later.

2. Prediction of thinning yield.

Since basal area yield estimation is calculated as the difference between two predictions, one before thinning and another after thinning, yield prediction can be expected to vary more widely than individual basal area prediction. The Sunny Corner thinning yield comparison showed that from 15 yield predictions 3 were within 2% of the measured yield, 11 within 5% and 14 were within 10%. The Green Hills thinning yield comparison showed that from 29 yield predictions 7 were within 2% of the measured yield, 12 within 5% and 21 within 10%.

Table 6. Summary of model testing. Before thinning (B.T.), after thinning (A.T.) and yield basal areas (m^2 ha^{-1}) expressed as a percentage comparison of model predicted to measured values. All predictions are made from the time of first thinning. Where the model has overpredicted, percentages are shown in bold type.

(a) For five thinning schedules from experiment BT11471, Sunny Corner State Forest.

			Percent basal area (predicted/measured)					
Thinning		Age (yr)	Thinning schedule 1A	1B	2A&B	3A	3B	Average
First	A.T.	14.3	**107**	**108**	**104**	**100**	**101**	**104**
Second	B.T.	18	**104**	**106**	**107**	**102**	**103**	**104**
	A.T.		**104**	**107**	**110**	**106**	**104**	**106**
Intermediate measurement		25	96	**100**	**104**	97	96	99
Clearfall		34	**103**	**107**	**114**	**104**	**103**	**106**
Average			**103**	**106**	**108**	**102**	**101**	**104**

		Percent basal area yield (predicted/measured)					
Thinning	Age (yr)	Thinning schedule 1A	1B	2A&B	3A	3B	Average
First	14.3	88	86	95	99	98	93
Second	18	**104**	**104**	**105**	95	**102**	**102**
Clearfall	34	**103**	**107**	**114**	**104**	**103**	**106**
Average		98	99	**105**	99	**101**	**101**

(b) For five thinning schedules from experiment AT11170, Green Hills State Forest.

			Percent basal area (predicted/measured)					
Thinning		Age			Thinning schedule			
		(yr)	1	2	3	4	5	Average
First	A.T.	15	103	107	102	106	104	104
Second	B.T.	17	99	102	100	102	105	102
	A.T.		99	100	100	102	103	101
Third	B.T.	20	102	96	96	97	103	99
	A.T.		100	98	99	96	102	99
Fourth	B.T.	24	94	-	93	91	99	94
	A.T.		96	-	94	93	100	96
Fifth	B.T.	27	100	95	96	93	98	96
	A.T.		108	96	93	90	97	97
Clearfall		32	117	108	104	93	99	104
Average	B.T.		102	100	98	95	101	99
	A.T.		103	100	98	97	101	100

		Percent basal area yield (predicted/measured)					
Thinning	Age			Thinning schedule			
	(yr)	1	2	3	4	5	Average
First	15	99	96	96	93	93	95
Second	17	99	106	101	104	113	105
Third	20	107	92	88	98	104	98
Fourth	24	83	-	74	72	86	79
Fifth	27	81	93	101	99	100	95
Clearfall	32	117	108	104	93	99	104
Average		98	99	94	93	99	96

3. Prediction of stand basal area yield over a full rotation.

For the five Sunny Corner thinning sequences, the average ratios of predicted over measured total rotation basal area yield were 98% and 99% (routine thinning), 105% (routine plus third row thinning) and 99% and 101% (third row thinning). Overall the yield was overestimated by 1%. For the five Green Hills thinning sequences, the ratios were 98%, 99%, 94%, 93%, and 101% in order of decreasing thinning intensity. Overall the yield was underestimated by 4%.

An advantage of this modelling method is that at the time of first thinning, predictions of yield for the whole rotation can be made (as in the example given). Thus the average clear felling yield for the five Sunny Corner thinning sequences was forecast 20 years ahead of time to within 6%, and within 4%, 17 years ahead of time for the five Green Hills thinning sequences.

However, as it is usual management practice to measure the stands at each thinning, progressive correction of the stand-segment basal areas will be possible as the rotation proceeds and hence less distant and more accurate yield prediction will result.

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APPENDIX 1 : Stand basal area prediction procedure using the model equations - an example.

A three step worked example is detailed showing the model prediction procedure to estimate the future growth and yield of a given stand for which future sequential thinning is proposed. The required inputs to the four model equations are taken from a measured test stand, so that the simulation of stand basal area growth, from first thinning through intermediate thinnings to clear felling can be demonstrated and the accuracy of prediction evaluated. The model prediction accuracy is tested by a comparison between the model predicted basal areas and the test stand measured basal areas.

The model input requirements are :

(i) The stand basal area immediately prior to first thinning;

(ii) A representative stand increment value and the age period over which it applied;

(iii) The number, residual stocking and the ages of the proposed thinnings.

The test stand basal area data are taken from a real forest situation where a measured stand growth sequence has included five thinnings carried out between the ages 15 to 32. The thinning sequence data shown below are the average of four replicates of thinning schedule 3, Experiment AT11170 in Green Hills State Forest, Albury Region, New South Wales.

Thinning number		Age (yr)	Stocking (trees ha^{-1})	Basal area (measured) (m^2 ha^{-1})
First	B.T.[1]	15	1386	37.90
	A.T.		489	18.31
Second	B.T.	17	489	24.31
	A.T.		333	18.31
Third	B.T.	20	333	26.76
	A.T.		212	18.06
Fourth	B.T.	24	212	27.71
	A.T.		193	25.38
Fifth	B.T.	27	193	30.74
	A.T.		109	20.34
Clearfall		32	109	26.22

---

[1] B.T. = Before Thinning; A.T. = After Thinning

Step 1. Determination of stand site potential category.

Depending on the stand information available, there are two methods to determine stand site potential.

(i) Inspection of Figure 3.

Given a representative stand increment and the mid-period age over which it applies, and provided that the stand stocking is high enough to produce plateau increment (above 700 trees ha^{-1} approximately prior to first thinning) the site potential category can be read from Figure 3.

The test stand measurements given here do not allow the calculation of plateau increment and hence site potential cannot be read from Figure 3.

(ii) Calculation of increment for critical points P and C with interpolation.

If thinning has taken place such that the stocking of the earliest calculated increment is below critical point P (as is the case here), the site potential category can be determined by comparing the observed representative stand increment with a table of the increment of critical points P and C for each site category (calculated from model equations I and II), and the linearly interpolated increment for the observed stocking of each site category.

Applying method (ii) to the Green Hills schedule 3 thinning sequence :

The measured increment I(S) over the growth period 15 to 17 years is 3.13 m^2 ha^{-1} yr^{-1} where

Mean age	A	=	16 years
Stocking	N = S[1]	=	489 trees ha^{-1}

From model II for mid-period age 16 years, the stocking of critical points P and C are respectively :

N(C) = 293 trees ha^{-1}

N(P) = 619 trees ha^{-1}

Thus the test stand has a stocking (489 trees ha^{-1}) between that of point P and point C at this age. The following values of I(C) and I(P) were then calculated for the five site categories from model I. By proportional linear interpolation, corresponding increment values were determined for the same site categories for this particular case where N = 489 trees ha^{-1} :

Site category	Basal Area Increment ($m^2 ha^{-1} yr^{-1}$) (Stocking trees ha^{-1}) I(C) (N=293)	 I(S[1]) (N=489)	 I(P) (N=619)
Q1	2.64	3.07	3.36
Q2	2.29	2.63	2.86
Q3	1.98	2.24	2.43
Q4	1.72	1.93	2.06
Q5	1.49	1.65	1.75

From inspection of the site category comparison table, the site category of the test stand with measured increment of 3.13 ($m^2 ha^{-1} yr^{-1}$) would thus most closely approximate that of category Q1.

Step 2. Determination of initial basal area for the selected stand-segments

The initial basal area ($BA_o(S)$) of selected stand-segments (i.e. future residual stockings following thinning) at first thinning can be estimated from model III. If, in addition to thinning from below, outrow thinning has occurred, an adjustment must be made.

For the test stand at first thinning, model inputs are as follows :

Age	A	=	15 years
Initial stocking	N	=	1386 trees ha^{-1}
Initial Basal Area	$BA_o(N)$	=	37.90 $m^2 ha^{-1}$

Routine eighth row outrow adjustment

i.e. n = 8

Adjust stand-segment stockings (S) by $n (n-1)^{-1}$, i.e. 8/7

Site potential category (Q1)

i.e. K = -0.5

Adjust predicted $BA_o(S)$ by $(n-1) n^{-1}$, i.e. 7/8

The stand-segment stockings are shown below.

From model III, the basal areas of the stand-segments (BA(S)) at age 15, i.e. the basal area of those stand-segments which will become residual stockings at later ages, are calculated to be as follows :

	Stand-segment stocking	Adjusted stand-segment stocking	Basal area	
			age 15 predicted	age 15 measured
	(trees ha^{-1})		(m^2 ha^{-1})	
S[1]	489	559	18.80	18.31
S[2]	333	381	13.96	
S[3]	212	242	9.63	
S[4]	193	221	8.93	
S[5]	109	125	5.50	

Hence, the measured basal area following first thinning at age 15 i.e. 18.31 m^2 ha^{-1} was overestimated by the model as 18.80 m^2 ha^{-1} (103%).

Step 3. Determination of basal area increment

(1) For the period following first thinning, i.e. 15-17 years, model inputs are as follows :

Mean age A = 16 years

Initial stocking N = S[1] = 489 trees ha^{-1}

Site potential category (Q1) i.e. K = -0.5

Substituting these values into model equation III below gives the following basal area for the stand-segments :

	Stand-segment stocking	Annual Increment I(S)	Period Increment I(S)p	Basal area		
				age 15 predicted	age 17 predicted	age 17 measured
	(trees ha^{-1})	(m^2 ha^{-1} yr^{-1})		(m^2 ha^{-1})		
S[1]	489	2.92	5.84	18.80	24.64	24.57
S[2]	333	2.19	4.38	13.96	18.34	18.31
S[3]	212	1.56	3.12	9.63	12.75	
S[4]	193	1.45	2.90	8.93	11.83	
S[5]	109	0.95	1.90	5.50	7.40	

Hence the measured basal areas before and after-thinning at age 17, i.e. 24.57 and 18.31 m^2 ha^{-1} are closely estimated by the model as 24.64 (100%) and 18.34 m^2 ha^{-1} (100%) respectively.

(2) For the period between 2nd and 3rd thinning, i.e.. 17 to 20 years, the procedure is the same :

Mean age	A	=	18.5 years
Initial stocking	N = S[2]	=	333 trees ha^{-1}
Site potential category (Q1)	i.e. K	=	-0.5

	Stand-segment stocking	Annual Increment I(S)	Period Increment I(S)p	Basal area age 17 predicted	Basal area age 20 predicted	Basal area age 20 measured
	(trees ha^{-1})	(m^2 ha^{-1} yr^{-1})		(m^2 ha^{-1})		
S[2]	333	2.43	7.29	18.34	25.63	26.76
S[3]	212	1.73	5.19	12.75	17.94	18.06
S[4]	193	1.62	4.86	11.83	16.69	
S[5]	109	1.05	3.15	7.40	10.55	

Hence the measured basal areas before and after-thinning at age 20, i.e. 26.76 and 18.06 m^2 ha^{-1} are slightly underestimated by the model as 25.63 (96%) and 17.94 (99%) m^2 ha^{-1} respectively.

(3) For the period between 3rd and 4th thinning, i.e. 20-24 years, calculate similarly :

Mean age	A	=	22 years
Initial stocking	N = S[3]	=	212 trees ha^{-1}
Site potential category (Q1)	i.e. K	=	-0.5

	Stand-segment stocking	Annual Increment I(S)	Period Increment I(S)p	Basal area		
				age 20 predicted	age 24 predicted	age 24 measured
	(trees ha^{-1})	(m^2 ha^{-1} yr^{-1})		(m^2 ha^{-1})		
S[3]	212	1.96	7.84	17.94	25.78	27.71
S[4]	193	1.82	7.28	16.69	23.97	25.38
S[5]	109	1.19	4.76	10.55	15.31	

The measured basal areas before and after thinning at age 24, i.e. 27.71 and 25.38 m^2 ha^{-1} are underestimated by the model as 25.78 (93%) and 23.97 m^2 ha^{-1} (94%) respectively.

(4) For the period between 4th and 5th thinning, i.e. 24 to 27, calculate similarly :

Mean age A = 25.5 years

Initial stocking N = S[4] = 193 trees ha^{-1}

Site potential category (Q1) i.e. K = -0.5

	Stand-segment stocking	Annual Increment I(S)	Period Increment I(S)p	Basal area		
				age 24 predicted	age 27 predicted	age 27 measured
	(trees ha^{-1})	(m^2 ha^{-1} yr^{-1})		(m^2 ha^{-1})		
S[4]	193	1.81	5.43	23.97	29.40	30.74
S[5]	109	1.18	3.54	15.31	18.85	20.34

The measured basal areas before and after thinning at age 27, i.e. 30.74 and 20.34 m^2 ha^{-1} are underestimated by the model as 29.40 (96%) and 18.85 m^2 ha^{-1} (93%) respectively.

(5) For the period between 5th thinning and clear felling, i.e. 27 to 32, calculate similarly :

Mean age A = 28.5 years

Initial stocking N = S[5] = 109 trees ha^{-1}

Site potential category (Q1) i.e. K = -0.5

	Stand-segment stocking	Annual Increment I(S)	Period Increment I(S)p	Basal area		
				age 27 predicted	age 32 predicted	age 32 measured
	(trees ha^{-1})	(m^2 ha^{-1} yr^{-1})		(m^2 ha^{-1})		
S[5]	109	1.41	7.05	20.34	27.39	26.22

The measured basal area prior to clear falling, i.e. 26.22 m^2 ha^{-1} is overestimated by the model as 27.39 m^2 ha^{-1} (104%).

A summary of model predicted and measured basal areas for a complete rotation of thinning schedule 3 is shown in Table A. The predicted values are shown to be close to the measured values for individual basal areas and yield.

Table A. Example of model working: comparison of model predicted and measured basal areas and basal area yields for before thinning (B.T.) and after thinning (A.T.) stand stockings, thinning schedule 3, experiment AT11170, Green Hills State Forest. All predictions are made from the time of first thinning (age 15).

Age		Stocking	Basal area			Yield		
			predicted	measured	%	predicted	measured	%
(yr)		(trees ha^{-1})	(m^2 ha^{-1})			(m^2 ha^{-1})		
15	B.T.	1386	-	37.90	-			
	A.T.	489	18.80	18.31	102	19.10	19.59	97
17	B.T.	489	24.64	24.31	100			
	A.T.	333	18.34	18.31	100	6.30	6.26	101
20	B.T.	333	25.63	26.76	96			
	A.T.	212	17.94	18.06	99	7.69	8.70	88
24	B.T.	212	25.78	27.71	93			
	A.T.	193	23.97	25.38	94	1.81	2.43	74
27	B.T.	193	29.40	30.74	96			
	A.T.	109	18.85	20.34	93	10.55	10.40	101
32	B.T.	109	27.39	26.22	104			
	Clearfall					27.39	26.22	104
					Total Yield	72.54	73.60	99

APPENDIX 2 : Data sets for model equations I and II

The data for model equations I and II are taken from Horne and Robinson (1988). They include values of basal area increment and residual stocking for both critical points C and P derived from Weibull increment / density relationships for each age period and site locality.

Forestry Region	Locality	Age Period	Increment (Stocking) C		Increment (Stocking) P	
		Years	$m^2 ha^{-1} yr^{-1}$	(trees ha^{-1})	$m^2 ha^{-1} yr^{-1}$	(trees ha^{-1})
Albury	Buccleuch I	10-14	3.03	(371)	3.62	(714)
		14-17	2.22	(266)	2.80	(597)
		17-20	1.72	(219)	2.28	(519)
	Buccleuch II	9-12	2.43	(326)	3.07	(659)
	Buccleuch III	10-13	2.98	(415)	3.70	(804)
		13-15	2.77	(398)	3.32	(762)
		15-18	2.28	(328)	2.80	(625)
	Red Hill	11-15	1.81	(428)	2.22	(797)
		15-18	1.74	(292)	2.09	(596)
		18-22	1.52	(297)	1.78	(623)
	Green Hills	12-15	3.06	(295)	3.56	(615)
		15-18	2.22	(226)	2.68	(501)
		18-21	1.78	(188)	2.43	(477)
Bathurst	Gurnang	12-15	2.50	(426)	3.38	(848)
		15-19	2.49	(407)	3.12	(847)
		19-23	2.06	(446)	2.52	(879)
		24-27	2.08	(345)	2.25	(679)
		27-29	2.00	(273)	2.03	(498)
		29-31	2.42	(326)	2.65	(662)
		31-33	2.21	(244)	2.58	(500)
		33-35	1.56	(213)	1.77	(492)
	Sunny Corner*	14-16	1.78	(358)	2.40	(851)
		16-18	2.36	(383)	2.84	(843)
		18-20	2.43	(319)	2.60	(613)
		20-25	2.63	(298)	3.12	(600)
		25-28	2.12	(310)	2.53	(630)
		28-32	1.68	(273)	2.15	(596)
	Vulcan	10-13	3.18	(411)	3.89	(842)
		13-15	2.47	(439)	3.24	(924)
Eden	Bondi	15-20	1.99	(266)	2.57	(584)
		20-23	2.03	(253)	2.46	(603)
		23-28	1.88	(219)	2.28	(482)

* The figure 25.3 years was used for age 25 calculations at Sunny Corner

APPENDIX 3 : Data sets for model equations III and IV

Data sets for model equations III and IV are available on 360 kilobyte 5.25" floppy disks from the authors. The data can be provided in one of two forms : (a) as ASCII files or (b) as PC SAS® (release 6.03) DATA SETS. As all analyses were performed with SAS using the default precision of 8 bytes per variable, some rounding errors will result from attempting to emulate reported results using the input ASCII data files.

Equation III data set

Nine experiments measured at a variety of ages, as low as 10.5 and as high as 37 years, are included in the data set. The experiments included are :

Experiment number	Experiment code	Location
AT11110	A	Red Hill
AT11122	B	Buccleuch II
AT11151	C	Buccleuch I
AT11152	D	Green Hills
ET11270	E	Bondi
BT11460	F	Gurnang
BT11470	G	Sunny Corner
AT12110	I	Buccleuch III
BT12420	J	Vulcan

The following variables are included in ASCII file EQ_III.DAT and SAS data set EQ_III.SSD :

Variable	Description	ASCII files		SAS data sets	
		Column range	FORTRAN Format	Variable Type	Variable Length
EXPT	Experiment number (experiment code in SAS data set)	1-7	A7	$	1
REP	Replicate number	8	A1	$	1
TRT	Treatment number	9	A1	$	1
A	Average Age for growth interval (yr)	10-13	F4.1	N	8
I_S	Stand-segment basal area periodic annual increment, I(S) (m^2 ha^{-1} yr^{-1})	14-19	F6.4	N	8
S	Stand-segment stocking (trees ha^{-1})	20-25	F6.1	N	8
N	Residual stocking (trees ha^{-1})	26-31	F6.1	N	8

An extract from the ASCII file EQ_III.DAT is shown as an example :

Column number :

```
         1         2         3
1234567890123456789012345678901
```

Variable* :

```
EXPT   RT   A   I_S    S     N
```

Example of data :

```
AT111101013.00.4487 100.01278.8
AT111101013.00.8148 200.01278.8
AT111101013.01.1085 300.01278.8
AT111101013.01.3644 400.01278.8
AT111101013.01.5833 500.01278.8
AT111101013.01.8082 600.01278.8
AT111101013.02.0003 700.01278.8
AT111101013.02.1622 800.01278.8
AT111101013.02.2929 900.01278.8
AT111101013.02.40361000.01278.8
AT111101016.50.5501 100.01267.8
```

* RT indicates variables REP and TRT. Control treatments are indicated by assigning value 0 to TRT.

Equation IV data set

Measurements between ages 10 and 20 years of unthinned plots in seven experiments planted at routine spacings (see Table 4) are included in the data set. One file has been created for each experiment. Files are named EQ_IV_x.DAT and EQ_IV_x.SSD where the value of x is the experiment code shown below. The experiments included are :

Experiment number	Experiment code	Location
AT11110	A	Red Hill
AT11122	B	Buccleuch II
AT11151	C	Buccleuch I
AT11152	D	Green Hills
ET11270	E	Bondi
BT11460	F	Gurnang
BT11470	G	Sunny Corner

The following variables are included in ASCII file EQ_IV_x.DAT and SAS data set EQ_IV_x.SSD :

Variable	Description	ASCII files		SAS data sets	
		Column range	FORTRAN Format	Variable Type	Variable Length
EXPT	Experiment number (experiment code in SAS data sets)	1-7	A7	$	1
PLOT	Plot number	8-12	A5	$	5
STRATUM	Stand-segment number*	13-14	I2	N	8
BA_S	Stand-segment basal area BA(S) (m^2 ha^{-1})	15-20	F6.3	N	8
BA_N	Stand basal area BA(N) (m^2 ha^{-1})	21-26	F6.3	N	8
S	Stand-segment stocking (trees ha^{-1})	27-32	F6.1	N	8
N	Residual stocking (trees ha^{-1})	33-38	F6.1	N	8
DATE	Date of measurement (SAS format MONYY.)	39-43	A5	N	4

* when stand-segments stockings are multiples of S_o, then $S = S_o$ x STRATUM

An extract from the ASCII file EQ_IV_A.DAT is shown as an example :

Column number :

```
         1         2         3         4
1234567890123456789012345678901234567890123
```

Variable*:

```
EXPT   PLOT ST  BA_S  BA_N   S     N  DATE
```

Example of data :

```
AT1111000011 1 3.15423.371  98.81284.3MAY55
AT1111000011 2 5.87223.371 197.61284.3MAY55
AT1111000011 3 8.32123.371 296.41284.3MAY55
AT1111000011 410.57023.371 395.21284.3MAY55
AT1111000011 512.62423.371 494.01284.3MAY55
AT1111000011 614.43323.371 592.81284.3MAY55
AT1111000011 716.14823.371 691.51284.3MAY55
AT1111000011 817.69823.371 790.31284.3MAY55
AT1111000011 919.16423.371 889.11284.3MAY55
AT11110000111020.53723.371 987.91284.3MAY55
AT11110000111121.75023.3711086.71284.3MAY55
```

* ST indicates variable STRATUM.