Modelling between-haul variability in the size selectivity of trawls

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Abstract

The selectivity of fishing gear can vary considerably from set to set even when deployment is replicated under controlled conditions. Analysis of data from size-selectivity experiments will be misleading if this between-deployment variability is not taken into consideration. Here, two approaches are presented for including the effect of between-haul variability when performing size-selectivity experiments for trawls. The first is a simple ad hoc approach that provides a ‘catch-all’ estimate of variability that also takes into account the effect of subsampling the catches. The second is a formal model of random between-haul variability that is fitted via maximum likelihood. The two approaches are applied to both covered-codend and twin-trawl selectivity experiments. The first method is seen to be more intuitive and interpretable, especially when selectivity may depend on catch size, but is limited in its ability to cope with complex experimental designs.

Keywords: Maximum likelihood; Overdispersion; Random effects; SELECT method

1. Introduction

Knowledge of the size (or age) selectivity of fishing gears is crucial for the management, monitoring and efficient exploitation of fisheries. For example, if the mortality of fish that escape the gear is low (relative to discard mortality) then an efficient gear will be one that releases most of the undersized individuals while retaining most of the larger ones. More generally, the retention properties of a fishing gear are quantified by a selectivity curve that gives the probability of retention as a function of fish size (or age).

Replicate tows of trawl gear typically show considerable between-haul variation in size selectivity (Fryer, 1991; Reeves et al., 1992). If this additional variation is not considered when analysing data from size-selectivity experiments then invalid conclusions may result. In particular, uncertainty in parameter estimates will be underestimated and spurious statistical significance is likely to be observed.

Here, we look at two different approaches to modelling between-haul variation when analysing data from size-selectivity experiments on trawl gear, and demonstrate these approaches on data from both covered-codend and twin-trawl experiments where the target species was school prawn (Metapenaeus macrolepis). The first approach applies an overdispersion correction to the selection curve that is obtained...
from fitting to the catch data summed over all hauls. This selection curve is the same as would be obtained if a single selection curve was simultaneously fitted to all the individual haul data. The fact that the combined-hauls fit is also the fit to the individual hauls permits a replication estimate of between-haul variation (REP) to be calculated.

The second approach is the formal modelling of between-haul variability using a hierarchical mixed-effects model (Fryer, 1991) to explicitly model variation in selectivity parameters between hauls. Here, we take advantage of the NLMIXED procedure available in Version 8 of SAS (SAS Institute, 1999). The NLMIXED procedure uses numerical integration to perform a maximum likelihood fit of the mixed-effects model simultaneously to all the individual haul data. In this regard it differs from the implementation employed by Fryer (1991) which requires the individual haul data to be modelled one haul at a time prior to fitting a mixed-effects model to the estimated selectivity parameters of each individual haul.

For paired-trawl experiments (e.g. alternate hauls, twin trawls, trouser trawls), the above approaches must be modified to include estimation of the relative efficiency parameter, p, of each haul (Millar and Walsh, 1992; Millar and Fryer, 1999). In particular, it is shown that the maximum likelihood estimate (MLE) of p can be calculated explicitly if the selection curve is logistic and this permits easy computation of REP for the combined-hauls fit to paired-trawl data. For the mixed-effects model approach, p can be modelled as an additional random effect.

It is worth noting that both approaches require the dataset or spreadsheet of catch data to be organised in the same way, notwithstanding that when subsampling occurs, the combined-hauls approach uses scaled catch data and the mixed-effects approach uses the unscaled data. Specifically, the data from the hauls should be "stacked" vertically. For example, the covered-codend example used below uses catch data from 19 hauls with 18 lengthclasses measured per haul. The data are organised in a dataset with 342 rows (equal to 19 \times 18) and three or more columns. The minimum set of columns would be lengthclass, codend catch and cover catch. Additional columns need to be added to include relevant covariates such as haul number, subsampling fraction (if only a sample of the catch is measured), gear type, catch weight, etc.

2. Methods

2.1. Combined-hauls fit applied to scaled individual-haul data

It is important to note that if subsampling of catches occurs then the scaled-up data are used because these provide an unbiased estimate of the true total catch when combined over hauls. However, scaling up of the measured catch data gives an inflated sample size that will result in an underestimate of uncertainty in the fitted selection curve. This underestimation can be severe if the sampling fractions are small. The REP estimate of overdispersion presented here provides a "catch-all" correction for the variability in the data. In addition to compensating for the effects of between-haul variation and of possible extra-Poisson variation in the measured catches, it also incorporates the effect of the inflated sample size if the measured data are scaled up.

The notation used below applies to both covered-codend and paired-trawl experiments. Specifically, for lengthclass \( l \) and haul \( h \), \( n_{lh}^s \) denotes the (scaled) catch in the experimental codend, \( n_{lh}^c \) denotes the (scaled) catch in the cover (if a covered-codend experiment) or in the control codend (if a paired-trawl type experiment), and \( n_{lh}^s = n_{lh}^c + n_{lh}^c \). The notation \( \hat{\gamma}_l^h \) is used to denote the proportion of the total catch (i.e. experimental codend plus cover/control) of lengthclass \( l \) fish that is expected to be taken in the experimental codend according to the given model of selectivity. The expected catch in the experimental codend is therefore given by \( n_{lh}^s \hat{\gamma}_l^h \).

Here, a single selection curve is fitted to the combined-hauls data, and then this curve is used to provide the expected catches for the individual hauls. In the case of covered codends, the \( \hat{\gamma}_l \) values are given by the selection curve fitted to the combined-hauls data, and the \( \hat{h} \) superscript can be omitted. However, in the case of paired trawls, these expected proportions also depend on the relative fishing efficiency (Millar and Fryer, 1999) of the experimental codend and control, and this will be permitted to vary between hauls. Consequently, \( \hat{\gamma}_l^h \) will depend on \( \hat{h} \) for paired-trawl data.

The replication estimate of overdispersion (REP) is given by evaluating how well the expected catch, calculated using the selection curve fitted to the combined-hauls data, fits the observed catches in each.
individual haul. That is, how well \( n_{i+h}^{\hat{h}} \) estimates \( n_{i+h} \). Specifically, REP (McCullagh and Nelder, 1989) is given by calculating the Pearson chi-square statistic for model goodness-of-fit and dividing by its degrees of freedom (d.o.f.). That is,

\[
REP = \frac{Q}{d}
\]

where \( d \) is the d.o.f. and

\[
Q = \sum_{i} \sum_{k} \left( \frac{n_{i+k}^{\hat{h}} - n_{i+k}^{h}}{n_{i+k}^{h}(1 - \hat{y})} \right)^2
\]

is the Pearson chi-square statistic and the summation is over all hauls and lengthclasses for which there was sufficient catch. (The model deviance could be used instead of the Pearson chi-square statistic and should give a similar estimate of REP.) The value of \( d \) is given by the number of terms in the summation minus the number of fitted parameters. It is common practice to restrict the summation to terms for which the expected catches in the codend and cover/control (\( n_{i+k}^{h} \) and \( n_{i+k}^{h}(1 - \hat{y}) \), respectively) are both greater than or equal to some value \( e \). This prevents over-inflation of \( d \). (We recommend that \( e \) should be at least 1, and prefer using \( e = 3 \) if the number of terms in (2) is not reduced too dramatically.)

Under the null hypothesis of no extra-Poisson variation, \( Q \) has an approximate chi-square distribution on \( d \) d.o.f. If this null hypothesis is rejected, standard errors of estimated parameters should be multiplied by \( \sqrt{REP} \).

In principle, fitting to the individual-hauls data as described above would permit the direct inclusion of covariates (e.g. gear type, catch weight). However, although REP is a meaningful and useful measure of total overdispersion, we do not feel that it should be used for formal model fitting. REP has a chi-square distribution when no extra-Poisson variation is present, but otherwise little else can be said about it because of the non-independence of catch data within each haul.

### 2.2. Mixed-effects analysis of individual-haul data

This approach provides a formal model for overdispersion due to between-haul variation, and assumes that the data from each individual haul are Poisson-distributed. Therefore, if catches are subsampled, it is the raw (unscaled) data that are used, and the effect of subsampling must be taken into account when fitting the model to these raw data (Millar, 1994; Appendix A).

The mixed-effects methodology incorporates the between-haul variation explicitly by letting each haul have its own set of parameters. For covered-codend experiments these parameters are just the parameters of the selection curve. In the case of paired trawls, the parameters also include the relative fishing efficiency of the experimental codend. The general notation used here will be to denote the set of parameters for haul \( h \) by \( \psi^h = (\psi^h_0, \ldots, \psi^h_N)^T \). For example, if fitting logistic selection curves to data from a covered-codend experiment then the curve can be specified by its length of 50% retention, \( L_{50} \), and selection range, \( SR \), in which case \( \psi^h = (L_{50}^h, SR^h)^T \). In the case of a paired-trawl experiment, \( \psi^h = (L_{50}^h, SR^h, p^h)^T \), where \( p^h \) is the probability that a fish entered the experimental codend of paired haul \( h \), given that it entered one or other of the codends.

The mixed-effects model of selectivity treats between-haul variation as a random effect and the parameters for each haul vary randomly about a mean vector of parameters. Controlled changes to the gear (e.g. changes in mesh size or shape) are also modelled, as fixed effects, by allowing the mean parameters to change in a systematic way. Specifically, it is assumed that the haul parameters vary from haul to haul according to the model

\[
\psi^h = \theta^h + \epsilon^h
\]

where \( \theta^h \) is the \( q \times 1 \) vector of expected parameter values for the selection curve of the gear used in haul \( h \) and \( \epsilon^h \) the \( q \times 1 \) random vector representing the random variation in the actual parameters for haul \( h \).

The \( \epsilon^h \) are assumed to be independent and multivariate normally distributed with zero mean and constant, but unknown, \( q \times q \) variance matrix \( D \). Thus, \( \psi^h \) has a multivariate normal distribution with mean \( \theta^h \) and variance \( D \). That is,

\[
\psi^h \sim N_q(\theta^h, D)
\]

Mixed-effects models can be very difficult to fit because calculating the likelihood function for the parameters of the fixed effects and \( D \) requires “averaging” over all possible values of the random-effects vectors, \( \epsilon^h \). This corresponds to a problem of
high-dimensional integration and is tractable only in special cases (such as linear mixed models with normal data). It is not tractable for selectivity data because the data are non-normal and the models are non-linear.

An alternative, which has been widely used in practice (Fryer, 1991; Reeves et al., 1992; Galbraith et al., 1994; Madsen et al., 2002) is to use the selection parameters and associated covariances estimated from each individual haul, and to model these according to a linear mixed model (Laird and Ware, 1982). Effectively, the "data" for each haul are reduced to the MLE $\hat{\theta}$ (and its covariance matrix), and Eq. (3) is fitted to these reduced "data" with $\theta$ required to be a linear function of gear covariates.

More recently, there is now software available that can fit more general forms of mixed models using numerical quadrature techniques to approximate the underlying high dimensional integrals. In particular, here we report on the application of SAS procedure NL MIXED (SAS Institute, 1999) for modelling between-haul variability in both covered-codend and paired-trawl experiments.

3. Application to covered-codend data

The catch data are from 19 covered-codend tows of a tow net with an experimental codend constructed from 30-mm knotted square mesh, done in the Clarence River, New South Wales, during July and August 2002. School prawns (M. maclurei) in the codend and cover were measured to the nearest millimetre carapace length, up to a maximum of approximately 250 animals each. For a full description of the methodology used during the experiment and a general overview of the Clarence River tow net fishery, refer to Macbeth et al. (2003) and Andrew et al. (1995), respectively.

For exploratory purposes, the individual-set data were modelled using individual logistic selection curves. These provided a good fit and lack of fit was not detected in any of the 19 individual fits. The $\text{i}_{\text{g}}$'s from the individual fits ranged from about 12 to 18 mm, and the selection ranges varied from about 2.5 to 10 mm (Fig. 1), notwithstanding that the higher estimated selection ranges also had high standard errors. In particular, for sets 10, 18 and 19, the standard errors of the estimated $\text{i}_{\text{g}}$ and SR were relatively high, due to very small catches. Plots of the individual-set estimates against codend catch weight suggest a weak relationship if any (Fig. 1).

3.1. Combined-hauls fit to individual-haul data

A single selection curve was fitted to the combined data (summed over hauls after scaling for subsampling of catches). The model deviance was 23.5 on 15 d.o.f. for a logistic selection curve, and 13.3 on 14 d.o.f. for a Richards selection curve. The model deviances from fitting to combined-hauls data will not satisfy the usual assumption of being approximately chi-square distributed, however, they still provide a clear quantification of relative lack of fit around the estimated combined-hauls selection curve. Deviance residuals from the logistic curve showed obvious lack of fit, which was not evident in the residual plot from the Richards fit. Hence, the Richards fit was used (Table 1).

A Richards selection curve was then fitted to the scaled individual-haul data (stacked vertically as explained in Section 1). These are exactly the same data as used in the combined-hauls fit, except that they are broken down by replicate haul, and consequently the fitted Richards curve and estimated standard errors were exactly the same as those obtained from the combined-hauls fit. The Pearson chi-square statistic from this fit was 993.2 on 99 d.o.f. when restricting the summation in Eq. (2) to terms with expected counts in experimental codend and cover of at least 3. This gives a REP of 10.03. Correcting for this amount of extra-Poisson variation, the standard errors estimated

<table>
<thead>
<tr>
<th>Method</th>
<th>Parameters</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combined hauls</td>
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<td></td>
</tr>
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<td>$l_{g}$</td>
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</tr>
<tr>
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<td>$h_{g}$</td>
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<tr>
<td></td>
<td>$l_{R}$</td>
<td>18.34</td>
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</tr>
<tr>
<td></td>
<td>SR</td>
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<td>0.85</td>
</tr>
<tr>
<td>Mixed model</td>
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<td>15.52</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>$\mu_{R}$</td>
<td>4.02</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>$\kappa_{R}$</td>
<td>1.66</td>
<td>0.57</td>
</tr>
</tbody>
</table>

*The combined-hauls selection curve is of Richards' form, and the standard errors of its estimated parameters have been multiplied by $\sqrt{\text{REP}} = 3.17$. 
from the fit were multiplied by a factor of $\sqrt{10.03} \approx 3.17$ (Table 1).

3.2. Mixed-effects model

A mixed-effects analysis was implemented using PROC NL MIXED. Logistic curves were used because they had adequately fitted the individual-haul data. Catch weight (kg) of school prawn in the codend was used as a covariate to model $l_{50}$ and SR,

$$l_{50} = \mu_{50} + \theta_1 w^h + \epsilon_{l1}$$

$$SR^h = \mu_{SR} + \theta_2 w^h + \epsilon_{SR^h}$$

where $w^h$ is log(catch weight + 1) of school prawns and $(\epsilon_{l1}, \epsilon_{SR^h})^T$ are bivariate normal with zero mean vector and $2 \times 2$ variance matrix $D$.

PROC NL MIXED would not converge for an arbitrary variance matrix $D$, but did converge when the covariance between $\epsilon_{l1}$ and $\epsilon_{SR^h}$ was assumed to be zero, and this fit had a log-likelihood of $-349.5$ (see Appendix A for the PROC NL MIXED code used to fit this model). A simpler model without the catch weight parameters $\theta_1$, $\theta_2$, and with $\sigma_{SR^h} = \text{Var}(\epsilon_{SR^h}) = 0$ (i.e. no between-haul variability in selection range) had log-likelihood of $-350.0$ and hence was preferable (Table 1).

4. Application to paired-trawl data

The data are catches of school prawns ($M. \text{macleayi}$) from 60 twin-trawl hauls conducted in March 2002
in Lake Woolooeyah in New South Wales (see Broadhurst et al., 2004). Twenty hauls used an experimental 40 mm knotted diamond-mesh codend. The other 40 hauls used experimental codends constructed from unknotted 20 mm square mesh, with 20 of these hauls using a tapered codend (tapering from 110 bar circumference to 54 bar at the end of the posterior section) and 20 using an untapered codend (110 bar circumference throughout). School prawns in both experimental codends and control were measured to the nearest millimetre carapace length, up to a maximum of 250 animals.

An exploratory look at the \( l_{50} \)'s and SRs from logistic selection curve fits to the 60 individual hauls suggests that the diamond-mesh codend may have a smaller \( l_{50} \) than the square-mesh codends. There is also some suggestion of \( l_{50} \) decreasing with catch weight in the untapered square-mesh codend (Fig. 2).

4.1. Combined-hauls analysis of individual haul data

Analysis of paired-trawl data includes an additional parameter, \( p \), the probability that a fish entering the paired trawl will enter the experimental codend. Here, a separate \( p^k \) for each haul was fitted.

Estimation of \( Q \) (Eq. (2)) was done in two steps. First, a selection curve, \( f(t) \), was fitted to the (scaled) data combined over all hauls. Second, using \( f(t) \), the MLE of \( p^k \) was calculated for each individual.
Reducing the discarding of small prawns

hails. When \( \hat{r}(l) \) is a selection curve of logistic form then the MLE of \( p^h \) can be obtained explicitly (Appendix B),

\[
p^h = \frac{\sum n^h_i}{\sum (n^h_i + n^h_{l2} \hat{r}(l))}
\]

This equation may not be exact for non-logistic selection curves, but will still provide an extremely accurate approximation that should be more than adequate in practice.

With \( \hat{p}^h \) calculated as above, the proportion of length \( l \) prawns that is expected to be caught in the experimental codend is given by (Millar and Walsh, 1992)

\[
\hat{y}^h = \frac{\hat{p}^h \hat{r}(l)}{\hat{p}^h \hat{r}(l) + (1 - \hat{p}^h)}
\]

from which \( Q \) can be calculated using Eq. (2). When calculating REP it is necessary to deduct an additional degree of freedom for each estimated \( \hat{p}^h \).

For the prawn data of Broadhurst et al. (2004), the combined-hauls data from the diamond and tapered square-mesh codends were well fitted by logistic selection curves and fits of Richards selection curves were almost identical. The logistic fit to the untapered square-mesh data showed some lack of fit, and when compared to the Richards curve, the difference in model deviance resulted in a \( p \)-value of 0.04 (notwithstanding that this \( p \)-value should be regarded as indicative only). Using the Bonferroni correction for three comparisons, a \( p \)-value must be less than 0.05/3 = 0.01667 to reject the null hypothesis at the 5% level, hence the logistic selection curve was used for all three codends. The \( Q \) values for the diamond, tapered square, and untapered square-mesh codends were 2274 on 266 d.o.f., 2886 on 261 d.o.f., and 2242 on 254 d.o.f., respectively. These correspond to REP values of 8.55, 11.06, and 8.83, respectively (Table 2).

Broadhurst et al. (2004) used the Wald statistic (Kotz et al., 1982) to test equality of the selection curves for the three gears. They found that the selection curve of the diamond-mesh codend was significantly different (\( p < 0.001 \)) from the selection curves of the square-mesh codends, and that the selection curves of the two square-mesh codends were not significantly different (\( p = 0.71 \)).

<table>
<thead>
<tr>
<th>Gear</th>
<th>Parameters</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 mm diamond</td>
<td>REP</td>
<td>8.55</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>( l_5 )</td>
<td>6.62</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>( l_{10} )</td>
<td>10.50</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>SR</td>
<td>3.88</td>
<td>0.59</td>
</tr>
<tr>
<td>20 mm tapered square</td>
<td>REP</td>
<td>11.06</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>( l_{5} )</td>
<td>8.52</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>( l_{10} )</td>
<td>10.10</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>( l_{15} )</td>
<td>11.68</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>SR</td>
<td>3.15</td>
<td>0.27</td>
</tr>
<tr>
<td>20 mm untapered square</td>
<td>REP</td>
<td>8.83</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>( l_{5} )</td>
<td>8.53</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>( l_{10} )</td>
<td>10.28</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>( l_{15} )</td>
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<td>0.37</td>
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<tr>
<td></td>
<td>SR</td>
<td>3.49</td>
<td>0.34</td>
</tr>
</tbody>
</table>

* All selection curves are logistic. The standard errors have been multiplied by \( \sqrt{REP} \).

4.2. Mixed-effects model

The mixed-effects Eqs. (4) and (5) were used to model \( \hat{y}^h \) and SR, respectively. In addition, the relative efficiency parameters were modelled as

\[
\logit(p^h) = \log\left(\frac{p^h}{1 - p^h}\right) = \mu_p + e_1^p
\]

where each \( e_1^p \) is normal with mean zero and variance \( \sigma^2_p \), and is assumed to be independent of \( e_1^h \) and \( e_2^h \).

PROC NLMIXED exhibited convergence problems when attempting to fit the model specified by Eqs. (4), (5) and (8) to the data from the three gears. Instead, the model was fitted separately to each gear. Once again, PROC NLMIXED would not converge when an arbitrary variance matrix \( \sum \) was specified, but did converge when the covariance between \( e_1^h \) and \( e_2^h \) was assumed to be zero. From the fits to the individual gears, it was found that the simpler model without the parameters \( \theta_1, \theta_2 \) and \( \sigma^2_{SR} = 0 \) (i.e. no between-haul variability in SR) was preferable (Table 3). This model had parameters \( \mu_{l_5}, \mu_{SR}, \mu_p, \sigma^2_{l_5} \), and \( \sigma^2_p \) (Table 3).

Further experimentation with PROC NLMIXED revealed that it was possible to fit the above model simultaneously to the data from all three gears while using separate \( \mu_{l_5} \) and \( \mu_{SR} \) for each gear and common \( \mu_p, \sigma^2_{l_5} \) and \( \sigma^2_p \) parameters for all three.

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Table 3
Forward selection of mixed-effects fits to the twin-trawl catches of prawn

<table>
<thead>
<tr>
<th>Gear</th>
<th>Parameters</th>
<th>Negative log-likelihood</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 mm diamond</td>
<td>(μ₀₀, μₘ₀₀, σ₀₀²)</td>
<td>701.59</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀²)</td>
<td>684.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂)</td>
<td>677.33</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂, σ₂₀)</td>
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<td>0.089</td>
</tr>
<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂, σ知晓)</td>
<td>675.25</td>
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<tr>
<td>20 mm tapered square</td>
<td>(μ₀₀, μₚ₀₀, σ₀₀²)</td>
<td>749.61</td>
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<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀²)</td>
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<td></td>
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<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂)</td>
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<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂, σ₂₀)</td>
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<td>0.038</td>
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<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂, σ知晓)</td>
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<td>0.657</td>
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<tr>
<td>20 mm untapered square</td>
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<td>692.57</td>
<td>&lt;0.001</td>
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<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂, σ₂₀)</td>
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<td>0.286</td>
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<tr>
<td></td>
<td>(μ₀₀, μₚ₀₀, σ₀₀², σ₀₂, σ知晓)</td>
<td>688.69</td>
<td>0.021</td>
</tr>
</tbody>
</table>

* Using the Bonferroni correction for testing across the three gears, a p-value of 0.0167 or less is required for a term to be considered significant at the 5% level. The tests for variance components are one-sided. The model selected as most appropriate for all three gears has parameters (μ₀₀, μₚ₀₀, σ₀₀², σ₀₂).

Table 4
Mixed-effects fits to the twin-trawl catches of prawn

<table>
<thead>
<tr>
<th>Gear</th>
<th>Parameters</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 mm diamond</td>
<td>μ₀₀</td>
<td>8.08</td>
<td>0.40</td>
</tr>
<tr>
<td>Both 20 mm square</td>
<td>μ₀₀</td>
<td>10.24</td>
<td>0.27</td>
</tr>
<tr>
<td>All gears</td>
<td>μₚ₀₀</td>
<td>3.48</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>σₚ₀₀²</td>
<td>0.0025</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>σₚ₀₀²</td>
<td>2.02</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>σ知晓²</td>
<td>0.038</td>
<td>0.01</td>
</tr>
</tbody>
</table>

gears, for a total of nine parameters. This model had a log-likelihood of -2054.65. A simpler model with common μ₀₀ for all three gears and a common μₚ₀₀ for the two square-mesh codends (a total of six parameters) reduced the log-likelihood by just 1.06 and hence this simpler model was preferred (Table 4). The parameter μₚ₀₀ was not statistically significant in this model, but we elected to retain it to avoid any bias that may arise if μₚ₀₀ were set equal to zero.

5. Discussion

Estimates of selectivity parameters can differ somewhat between the two methods demonstrated here. This is not unexpected because the two methods are modelling different things. The combined-hauls approach is giving a simple quantification of "fisheries" selectivity, in the sense that it is an appropriate estimate of the contact-selection curve (Millar and Fryer, 1999) relevant to the fishery. This interpretation assumes that, when used in the fishery, deployment of the experimental codend and catch sizes will be similar to that experienced during the selectivity study. This selection curve would be suitable for use in studies of incidental mortality, say. The mixed-model approach is describing the structure and variability in individual hauls, and parameters such as μ₀₀ and μₚ₀₀ are not directly relevant to the selectivity of the experimental gear on the fishery. The mixed-model approach provides a formal method for modelling the effect of covariates. These could be observed covariates such as catch weight, or design covariates such as mesh size, hanging ratio, etc.

In the covered-codend example the value of SR estimated from the combined-hauls approach was substantially higher than the mean SR value estimated by the mixed-effects approach (Table 1). This is perhaps not surprising because between-haul variability tends to smear the effect of individual-haul selectivity. It was also the case that the estimated l₅₀ and
$\mu_{0b}$ differed noticeably in the covered-codend experiment (16.17 and 15.52 mm, respectively) and for the diamond-mesh codend in the twin-trawl experiment (8.56 and 8.08 mm, respectively). This may be due to the fundamentally different way in which the two approaches statistically weight the data from each haul.

The combined-hauls approach uses the scaled data and hence has the appealing property of weighting the data from each haul according to the catch weight in that haul. The more formal mixed-effects approach models the raw data and hence weights each haul equally (assuming sufficient catch to meet the sampling requirements). For the covered-codend data, it was noticed that the smallest two sampling fractions occurred for the catches in the covers in sets 1 and 5. These two sets had the largest $I_{b0}$'s of the 19 sets (Fig. 1). The combined-hauls analysis would have given these catches higher statistical weight than the mixed-effects analysis, resulting in a higher estimated $I_{b0}$. The above paragraphs highlight the need to understand the underlying differences between the combined-hauls and mixed-effects approaches, and to make an informed choice of which method to use. This choice may well depend on the objectives of the study and the intended use of the estimated selection curve(s). If the selectivity experiment involves a single gear then it may be enough to use the straightforward combined-hauls approach in conjunction with the calculation of REF.

If the selectivity experiment involves multiple gears then the choice between combined-hauls analysis or mixed-effects analysis may not be as easy. Fitting the mixed-effects model using software such as PROC NLMIXED is a more rigorous approach, but does require more assumptions regarding the form of the model. If selectivity of individual hauls is dependent on catch size then the concept of a mean $I_{b0}$ and $SR$ makes little sense and it is vital that the mixed-effects model includes catch weight as a covariate so that the above mean parameters are explicitly modelled as a function of catch weight. Unfortunately, we found here that the data did not permit PROC NLMIXED to fit models with many parameters or any covariance terms, and the simultaneous modelling of several gears and associated covariates may not always be possible using this software.

An alternative implementation of the mixed-effects methodology is to use the linear mixed-effects approach employed by Fryer (1991). This requires maximum likelihood fits to the data from each individual haul and this can be problematic, particularly when some hauls have limited data. For paired trawls this does have the advantage of estimating $p^b$ at the individual haul level and thereby avoiding the need to model $p^b$ in the mixed-effects model. However, this approach relies crucially on the assumption that the estimated selectivity parameters from the individual fits are (approximately) normally distributed with covariance matrix well estimated from the asymptotic covariance matrix. Like the mixed-effects approach used herein, this approach does not weight individual hauls according to the size of catch, and one would need to include catch weight as a covariate if it has an effect on selectivity (e.g. Madsen et al., 2002).

Acknowledgements

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Appendix A

SAS code for the mixed-effects fits to the covered-codend and paired-trawl data. Variable haul is the haul number and variables lenclass, wgt, q1, q2 are the length class, log(catch weight + 1) in codend, and sampling fractions of the experimental codend and cover/control, respectively. For covered-codend data, cover is the catch in the cover, and for paired-trawl data, control is the catch in the control codend. Variable codend contains the catch in the experimental codend.

A.1. Covered codend

```
PROC NLMIXED DATA=CoveredCodend
METHOD=GAUSS ABSGCONV
=0.000001[3];
total=codend+cover;
```
work=(q1/q3)*exp(2*ln(3)) * (lenclass- (meanL50+L50 +theta*lwt)) / (meanSR+SR+theta2*wgt));
q=r/work/(1+work);
PTRANS meanL50=15 meanSR=3 theta =0 theta2=0;
MODEL codend ~ BINOMIAL(total, r);
RANDOM L50 SR ~ NORMAL([0,0], [varL50,0, varSR]) SUBJECT=haul;
RUN;

A.2. Paired trawl

Here, Sg is an indicator variable that takes the value 1 for the tapered or untapered square-mesh codends, and 0 otherwise.

PROC NLMIXED DATA=TwinTrawl METHOD
=GAUSS ABSCONV=0.0000013;
total=codend+control;
work=exp(2*ln(3)) * (lenclass- (meanL50+L50 +Sg*DSqL50)) / (meanSR));
r=q/work/(1+work);
pets=meanPeta+p;
psplit=exp(pets)/(1+exp(pets));
phi=q1*psplit+r +q2*(1-psplit));
PTRANS meanL50=8.2 DSqL50 =1.7 meanSR=4.0 meanPeta
=0 varL50=2 varP=1;
MODEL test ~ BINOMIAL(total,phi);
RANDOM L50 P ~ NORMAL([0,0], [varL50,0, varP]) SUBJECT=haul;
RUN;

Appendix B

The log-likelihood function for paired-trawl data is

$$\log(L) = \sum_i \left\{ n_{1i} \log \left( \frac{pr(i)}{(1-p) + pr(i)} \right) + n_{2i} \log \left( \frac{1-p}{(1-p) + pr(i)} \right) \right\}$$

If the selection curve is logistic then it can be written as

$$r(i) = \frac{e^{a+bL}}{1 + e^{a+bL}}$$

and (after a bit of calculus),

$$\frac{\partial \log(L)}{\partial a} = \sum_i \left\{ n_{1i} (1-p) - n_{2i} pr(i) \right\} \frac{(1-r(i))}{(1-p) + pr(i)}$$

$$= \sum_i \left\{ n_{1i} (1-p) - n_{2i} r(i) \right\} \frac{(1-p) + pr(i) - r(i)}{(1-p) + pr(i)}$$

$$= \sum_i \left\{ n_{1i} + n_{2i} r(i) - \frac{n_{2i} r(i)}{(1-p) + pr(i)} \right\}$$

At the MLE the above derivative is zero, giving

$$\sum_i \left( n_{1i} + n_{2i} r(i) \right) = \sum_i \frac{n_{1i} + n_{2i} r(i)}{(1-p) + pr(i)} \quad \text{(B.1)}$$

Differentiating $\log(L)$ with respect to $p$, and using (B.1), gives

$$\frac{\partial \log(L)}{\partial p} = \frac{1}{p(1-p)} \sum_i \left\{ n_{1i} (1-p) - n_{2i} pr(i) \right\}$$

$$= \frac{1}{p(1-p)} \sum_i \left\{ n_{1i} - n_{2i} r(i) \right\}$$

$$= \frac{1}{p(1-p)} \sum_i \left\{ n_{1i} - p(n_{1i} + n_{2i} r(i)) \right\}$$

Setting this derivative to zero gives the result that the maximum likelihood estimate of $p$ is

$$\hat{p} = \frac{\sum_i n_{1i}}{\sum_i (n_{1i} + n_{2i} r(i))}$$

References


